Written Test No.2

November 3, 2022

Surname **Name** **Student's code.:**

There is no need
indeed, it is NOT allowed to use a programmable calculator!

Section 1: TRUE/FALSE QUIZZES

1. The Laplace transform of the derivative is equal to the transformed function multiplied by the Laplace abscissa s plus the value of the function in the time domain at the state 0 (initial condition)

   true ☐ false ■

2. In a second order system the overshoot is the fractional deviation of the output from its final value, at a peak of an oscillatory step response.

   true ■ false ☐

3. A system is BIBO stable if and only if all its poles pi have negative real parts, Re{pi} < 0.

   true ■ false ☐

4. The integral time is the constant that divides the integral of the error signal in the PID controller

   true ■ false ☐

Section 2: QUIZZES

1. If τ is the characteristic time of a self-regulating first order system to which an input step test is applied, the 99% of the new steady state value for the output is reached after at least:
   a. ☐ τ
   b. ☐ $3\ \tau$
   c. ■ $5\ \tau$
   d. ☐ $7\ \tau$

2. The PID controller transfer function is

   a. ☐ $G_c=K_c[1 + \tau_D\ s + \tau_I s]$
   b. ■ $G_c=K_c[1 + \tau_D\ s + 1/(\tau_I s)]$
   c. ☐ $G_c=K_c[1/(1+\tau_I s)+ \tau_D\ s]$
   d. ☐ $G_c=K_c/((\tau_{I+}\tau_D)\ s)$

Section 3: REFERENCE DYNAMIC MODELS

3.1. Response of a dynamic model

A thermometer with first-order time constant of 0.1 min and gain of 1.0 is in equilibrium with a liquid bath at $T_{ext_{ss}} = T_{m_{ss}} = 25°C$. At the time *t=0* min, the temperature of the bath $T_{ext}(t)$ is increased linearly at a rate of 1°C/min.

$$T_{ext}(t) = 25\,[°C] + 1\left[\frac{°C}{min}\right]\,t\,[min]$$

1. Write the forcing function $T_{ext}(t)$ in terms of deviation variable(s) $T'_{ext}(t)$.
2. Write the forcing function $T'_{ext}(t)$ in the Laplace domain ($\hat{T}_{ext}(s)$).
3. Obtain the expression, in the Laplace domain, of the measured thermometer temperature $\hat{T}_m(s)$.
4. Obtain the expression of the time evolution of the measured thermometer temperature $T_m(t)$
5. What is the difference between the measured temperature $T_m(t)$ and the bath temperature $T_{ext}(t)$ at:
    a. t = 0.1 min;
    b. t= 1.0 min.

t	*exp(-t)*
0	1.00
0.25	0.78
0.5	0.60
0.75	0.47
1	0.37
1.5	0.22
2	0.14
2.5	0.08
3	0.05
3.5	0.03
4	0.02
≥ 4.5	0

Hints:

- The Laplace transform of a ramp function is: $\mathcal{L}\{t\} = 1/s^2$
- The provided table can be used to approximate the exponential decay function:

Solution

1. Write the forcing function $T_{ext}(t)$ in terms of deviation variable(s) $T'_{ext}(t)$.

$$T'_{ext}(t) = 1\left[\frac{°C}{min}\right]\,t\,[min]$$

2. Write the forcing function $T'_{ext}(t)$ in the Laplace domain ($\hat{T}_{ext}(s)$).

$$\hat{T}_{ext}(s) = 1/s^2$$

3. Obtain the expression, in the Laplace domain, of the measured temperature $\hat{T}_m(s)$.

$$\mathrm{G(s)} = \frac{\hat{T}_m(s)}{\hat{T}_{ext}(s)} = \frac{1}{0.1s+1}$$

$$\hat{T}_m(s) = \mathrm{G(s)}\hat{T}_{ext}(s)$$

$$\hat{T}_m(s) = \frac{1}{0.1s+1}\cdot\frac{1}{s^2}$$

4. Obtain the expression of the time evolution of the measured temperature $T_m(t)$

Eq. 2.28 "Table of Laplace Transforms" in "Reference Tables for students"

2.28	$\frac{\alpha}{s^2(s+\alpha)}$	$t-\frac{1}{\alpha}[1-\exp(-\alpha t)]$

$$\hat{T}_m(s) = \frac{10}{0.1s \cdot 10 + 10} \cdot \frac{1}{s^2} = \frac{10}{s^2(s+10)}$$

$$\mathcal{L}^{-1}\{\hat{T}_m(s)\} = T'_m(t) = t - \frac{1}{10}[1-\exp(-10t)]$$

$$T_m(t) = 25 + t - \frac{1}{10}[1-\exp(-10t)]$$

5. What is the difference between the measured temperature $T_m(t)$ and the bath temperature $T_{ext}(t)$ after the change in temperature at:

$$T_m(t) - T_{ext}(t) = -\frac{1}{10}[1-\exp(-10t)]$$

   a. t = 0.1 min;

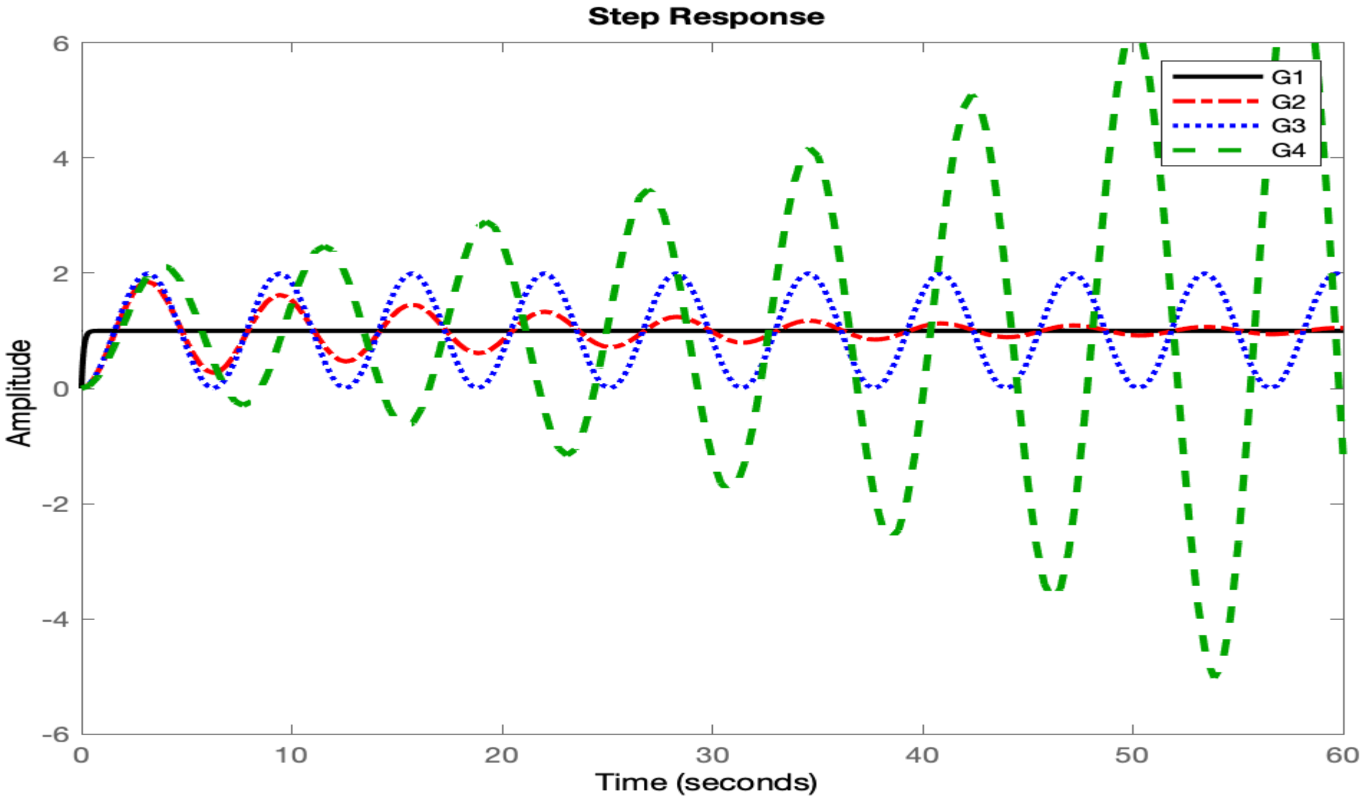
$$T_m(t = 0.1min) = -\frac{1}{10}[1-\exp(-10 \cdot 0.1)] = -0.063°C$$

   b. t= 1.0 min.

$$T_m(t = 1min) = -\frac{1}{10}[1-\exp(-10 \cdot 1)] = -0.1°C$$

3.2 Stability of dynamic system

The diagram reports the step responses of 4 linear dynamical systems



a. provide the definition of **BIBO stability**

A linear dynamic system is defined as stable if its response (output) is mathematically bounded for every input that is bounded, whatever its initial condition.

A system is (asymptotically) stable **if all of its poles have negative real parts.**

A system is unstable if at least one pole has a positive real part.

A system is **marginally stable** if it has one or more single poles on the imaginary axis and any remaining poles have negative real parts

b. Discuss which system is BIBO stable and which one BIBO unstable,

G1, G2 are BIBO stable

G3 is marginally stable

G4 is BIBO unstable

c. try to guess qualitatively the type of transfer function for G1, G2, G3, and G4

G1 **first order**;

G2 second order **underdamped**

G3 second order **undamped**

G4 second order **with positive poles**

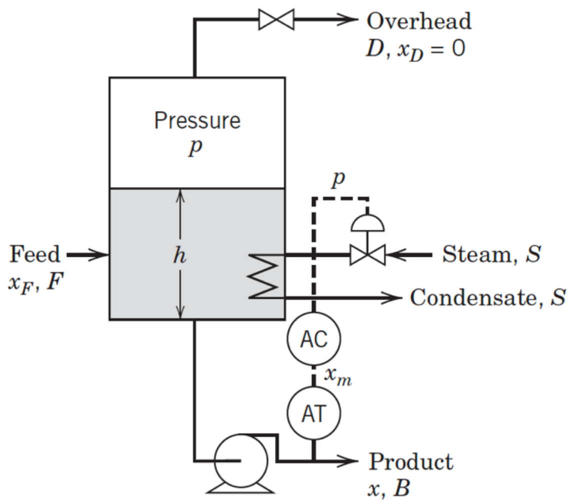
Section 4: CONTROL AND MONITORING

4.1. The feedback control

The figure shows a practical application of feedback control. In particular, a steam-heated evaporator used to concentrate a feed stream by evaporating water is shown.

Among the various process variables (flow rate, etc.)

1. select the **measured variable**
   Outlet concentration
2. select the **controlled variable**
   Outlet concentration
3. select the **manipulated variable**
   Steam flow rate
4. select the **disturbance variable** (if any)
   Feed concentration



Among the various process **block components** (tank, valves, pump, etc.)

5. select the **sensor/measuring device** AT
6. select the **comparator** AC
7. select the **actuator** Valve actuator
8. select the **final control element** Valve
9. what type of signal is used in the **control loop?** Electrical
10. what is the role of the tank in the **control loop system?** Process

***Note*:** The symbols in the picture are reported according to the standard instrumentation symbols published by the Instrumentation, Systems and Automation (ISA) Society. "A" as first letter means "Analysis"

Section 5: CONTROLLERS

5.1 Tuning the PID controller

An unknown process is stimulated, at time 0 s, in the set-point by a step function (red dotted line in the attached figure) and, in the open loop configuration, the response of the process variable (to be controlled at closed loop) is recorded (blue solid line in the attached figure).

1. From the dynamic response determine the value of the **dead time** t_d

t_d=3 s

2. Obtain the transfer function of an FOPDT fitting model for which K_p=1 and τ_p=1 s;

$$G(s) = \frac{1}{s+1} e^{-3\,s}$$

3. Describe in a few words the meaning of the FOPDT model

A true process model is usually neither first-order nor linear. Only the simplest processes exhibit such ideal dynamics. Therefore, in order to account for higher-order dynamics that are neglected in a first-order model, a time-delay term can be included in a first-order model. This modification can improve the agreement between model and experimental responses.

Open loop tuning methods are used to find the optimal PID controller parameters and the closed loop system response is recorded and reported in the attached figure.

4. Which is the best, according to the provided figure, tuning algorithm? (A qualitatively but detailed answer is required)

The Cohen and Coon algorithm is the best algorithm.
The IMC aggressive produce high overshoot and it does not reach the new steady state value in the time range recorder.
The IMC conservative is to sluggish, reaching values close to the new steady state value at the end of the recorded time frame.
The Cohen and Coon algorithm is quick enough to bring the system in the closed loop configuration at the new steady state value before the other two tested methods.

5. According to the best tuning algorithm proposed by you in 4), calculate the tuning parameters of the PID controller.

From the Cohen and Coon PID tuning ("PID_formule_tuning.pdf" in "Reference Tables for Students")

Controllore PID

Guadagno del controllore

$$K_C := \frac{\tau_P}{K_P \cdot t_d} \cdot \left(\frac{4}{3} + \frac{t_d}{4 \cdot \tau_P} \right)$$

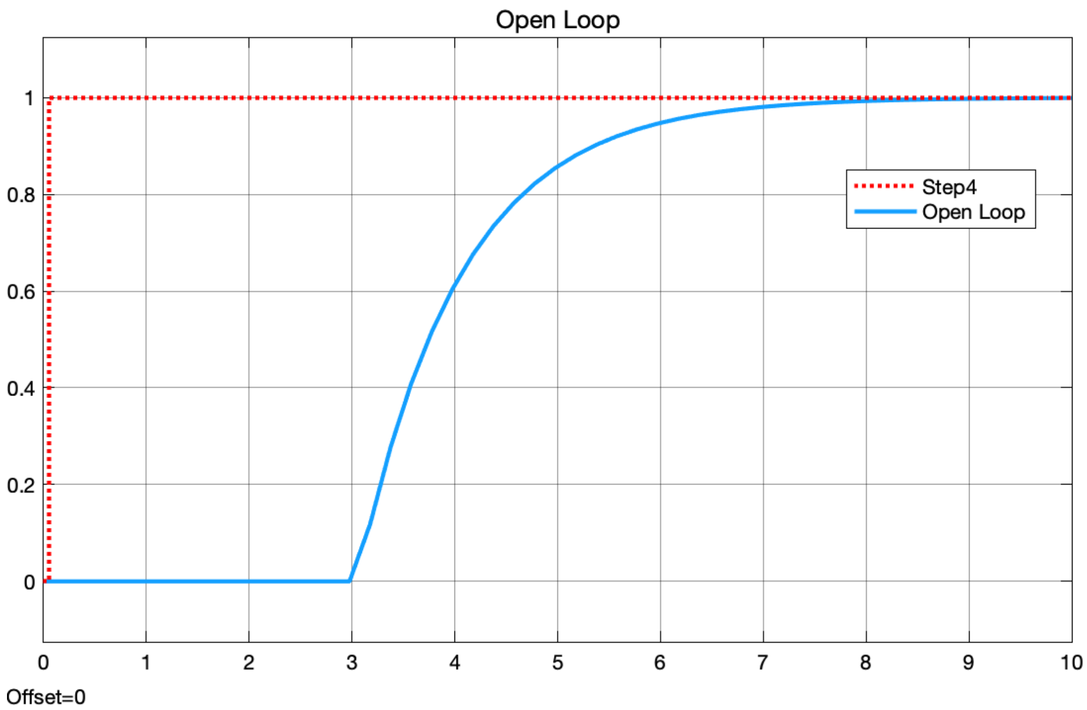
Reset time del controllore

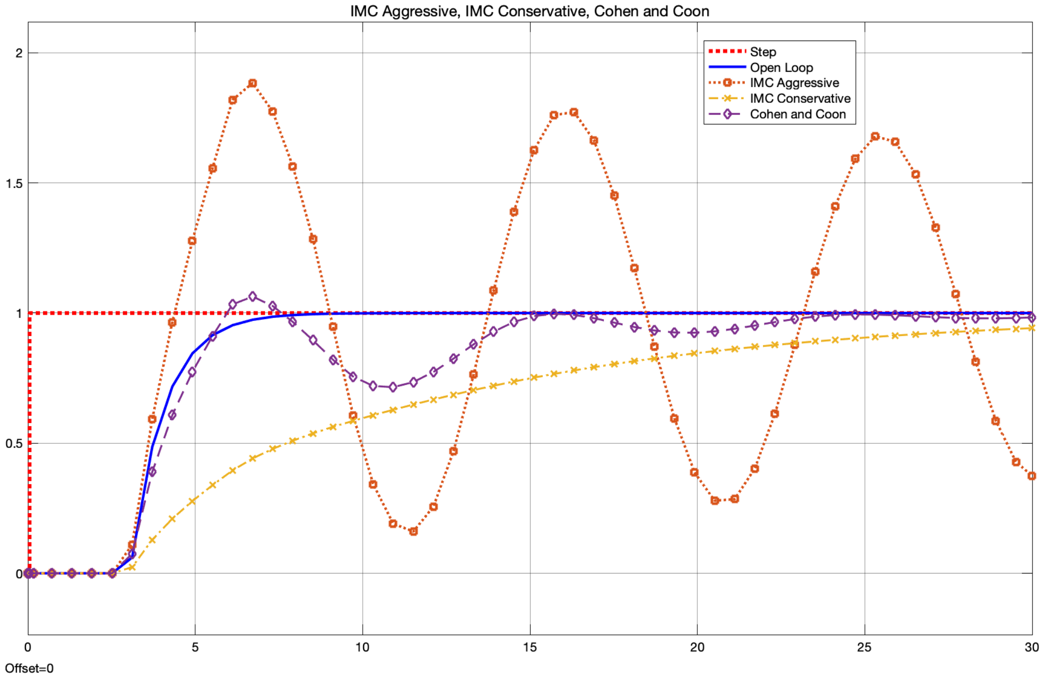
$$\tau_I := \frac{32 + 6 \cdot \frac{t_d}{\tau_P}}{13 + 8 \cdot \frac{t_d}{\tau_P}} \cdot t_d$$

tempo derivativo del controllore

$$\tau_D := \frac{4}{11 + 2 \cdot \frac{t_d}{\tau_P}} \cdot t_d$$

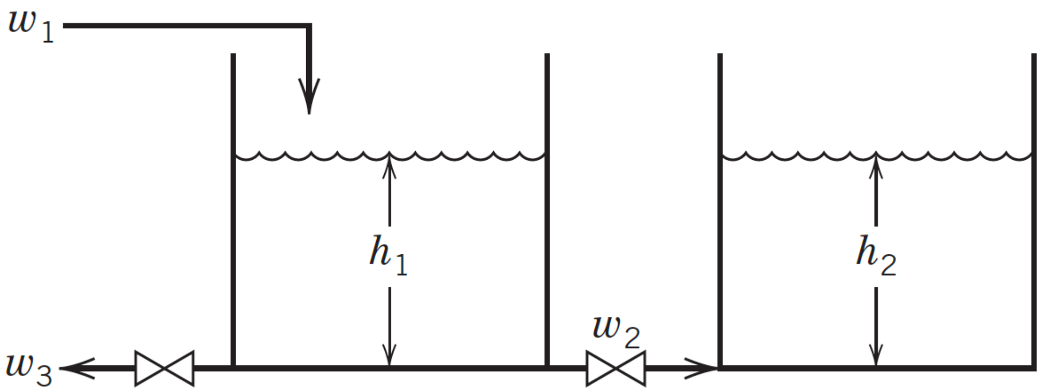
$$K_c = 0.6944$$
$$\tau_I = 4.054$$
$$\tau_D = 0.7059$$




IMC Aggressive, IMC Conservative, Cohen and Coon
Step
Open Loop
IMC Aggressive
IMC Conservative
Cohen and Coon
2
1.5
1
0.5
0
0
5
10
15
20
25
30
Offset=0

Section 6: MATHEMATICAL MODELLING

Two tanks are connected together in the following unusual way:



- The density of the liquid, ρ, is constant.
- The cross-sectional areas of the two tanks are A1 and A2.
- The two valves are linear with resistances $R_{v2}=R_{v3}=R_v$.

You must:

1. write the **dynamical model** of the system;

Since no indications were given for the flow rates, the assumption of mass flow rate will be used. Moreover the unit of measurements of the valve linear resistance R_v with flow rate were not specified. We can assume that it is referred to mass flow rate, so that $R_v = \left[\frac{s\,m}{kg}\right]$. An equivalent reasoning, consistent in terms of unit of measurements, would be legit.

During the exam, if you have doubts, please ask!

$$\begin{cases} \rho A_1 \dfrac{dh_1}{dt} = w_1 - \dfrac{h_1 - h_2}{R_v} - \dfrac{h_1}{R_v} \\ @t = 0, h_1 = h_{1_{ss}} \\ \rho A_2 \dfrac{dh_2}{dt} = \dfrac{h_1 - h_2}{Rv} \\ @t = 0, h_2 = h_{2_{ss}} \end{cases}$$

2. write the **steady state** model of the system;

$$\begin{cases} 0 = w_{1_{ss}} - \dfrac{h_{1_{ss}} - h_{2_{ss}}}{Rv} - \dfrac{h_{1_{ss}}}{R_v} \\ 0 = \dfrac{h_{1_{ss}} - h_{2_{ss}}}{Rv} \end{cases}$$

$$\begin{cases} \boldsymbol{w_{1_{ss}} = \dfrac{h_{1_{ss}}}{R_v}} \\ \boldsymbol{h_{1_{ss}} = h_{2_{ss}}} \end{cases}$$

3. list **input**, **state**, **output** variables and the **parameters** of the model;

input: $w_1(t)$
state: $h_1(t), h_2(t)$
output: $w_3(t) = \frac{h_1(t)}{R_v}$
parameters: ρ, A_1, A_2, R_v

4. is the dynamical model a linear model? If not, **individuate and linearize the non-linear term(s);**
   Linear
5. **write the model in the Laplace domain;**

$$\begin{cases} \rho A_1 \hat{h}_1 s = \widehat{w}_1 - \dfrac{\hat{h}_1 - \hat{h}_2}{Rv} - \dfrac{\hat{h}_1}{R_v} & (1) \\ \rho A_2 \hat{h}_2 s = \dfrac{\hat{h}_1 - \hat{h}_2}{Rv} & (2) \end{cases}$$

6. **obtain the transfer function** describing the evolution of $h_1(s)$ with respect to the input variable $w_1(s)$;

   The transfer function describing the evolution of $\hat{h}_1$(s) with respect to the input variable $\widehat{w}_1(s)$ can be obtained from eq. (1) of the previous step. However, the state variable $\hat{h}_2$ has to be reformulated in terms of $\hat{h}_1$(s) using equation (2).

$$\hat{h}_2 = \frac{\hat{h}_1}{\rho A_2 R_v s + 1} \qquad (3)$$

   Substituting (3) in (1):

$$\rho A_1 \hat{h}_1 s = \widehat{w}_1 + \frac{\hat{h}_1}{\rho A_2 R_v s + 1} \frac{1}{Rv} - 2\frac{\hat{h}_1}{R_v}$$

$$\dots$$

$$\frac{\hat{h}_1}{\widehat{w}_1} = \frac{R_v(\rho A_2 R_v s + 1)}{A_1 A_2 \rho^2 R_v^2 s^2 + s(A_1 R_v \rho + 2\, A_2 R_v \rho) + 1}$$

7. **classify the obtained transfer function** and individuate the parameters.

   Second order transfer function (second order equation in s at the denominator) with numerator dynamics (a zero is present).

   Gain=R_v (it can be calculated setting s=0, which correspond to $t \to \infty$)

$$\tau^2 = \sqrt{A_1 A_2 \rho^2 R_v^2}$$

$$2\xi\tau = (A_1 R_v \rho + 2\, A_2 R_v \rho)$$