Surname **Name** **Student's code.:**

There is no need
Indeed, it is NOT allowed to use a programmable calculator!

Section 1: TRUE/FALSE QUIZZES

1. The time constant τ_P of a first order system is the time interval which elapses before the step response reaches the 50 % of its final steady-state value

   true ☐ false ☑

2. A system is said to be BIBO stable if its output remains bounded for any bounded input.

   true ☑ false ☐

3. The steady-state error of a feedback-controlled system is the difference between the desired output and the actual output as time goes to infinity.

   true ☑ false ☐

4. A transfer function represents how a linear system modifies the Laplace transform of the input to produce the Laplace transform of the output.

   true ☑ false ☐

Section 2: QUIZZES

1. An industrial controller can be : (NB: mark only the wrong answer!)

a. ☐ "direct action"
b. ☐ "reverse action"
c. ☐ relay
d. ☑ FOPDT

2. The offset is:

a. ☐ $y_\infty - y_{SP}(t)$
b. ☑ $y_{SP}(t) - y_\infty$
c. ☐ $y_\infty - y_m(t)$
d. ☐ always positive

3. Which of the following "parameters" is not included in the 2nd order system law?

a. ☐ Process gain
b. ☑ *overshoot*
c. ☐ Natural oscillation period
d. ☐ Damping factor

Section 3: REFERENCE DYNAMIC MODELS

3.1. Response of a dynamic model

A chemical reaction is taking place in a tank, and the concentration of a reactant is being monitored by a concentration analyzer. The relationship between the measured concentration $C'_m(s)$ and the actual concentration $C'(s)$ is given by the following transfer function (in deviation variable form):

$$\frac{C'_m(s)}{C'(s)} = \frac{1}{s+1}$$

The system is at its steady-state (SS) value, with actual and measured concentration of 2 mol/L: C_{ss}= $C_{m_{ss}}$= 2 mol/L. A warning light on the analyzer turns on whenever the measured concentration drops below 1 mol/L. Suppose that at time t = 0 min, the concentration of the reactant in the tank begins to decrease, $C(t) = 2 - 0.5t$, where C has units of mol/L and t has units of minutes.

1. Which type of reference dynamic model is actually represented by the above transfer function?
2. How much is the steady-state gain?
3. How much is the time constant?
4. Is the process affected by delay? If so, how much is the time delay?
5. Write the forcing function $C(t)$ in terms of deviation variable(s) $C'^{(t)}$.
6. Write the forcing function $C'^{(t)}$ in the Laplace domain ($\hat{C}(s)$).
7. Obtain the expression, in the Laplace domain, of the measured concentration $\hat{C}_m(s)$.
8. Obtain the expression of the time evolution of the measured concentration in terms of deviation variable C'_m(t)
9. At what time, with the approximation of ±1 min, will the warning light turn on?

Hints:

- The following table can be used to approximate the exponential decay function:

t	*exp(-t)*
0	1.00
0.25	0.78
0.5	0.61
0.75	0.47
1	0.37
1.5	0.22
2	0.14
2.5	0.08
3	0.05
3.5	0.03
4	0.02
4.5	0.01

1) The transfer function represents a first-order system.
2) The steady-state gain is 1 (@s->0)
3) The time constant is 1.
4) The process is not affected by delay.
5) C(t) = 2 - 0.5t can be written in deviation variable form as C'(t) = -0.5t.
6) Taking the Laplace transform of $C'(t)$, we get $C'(s) = -\frac{0.5}{s^2}$.
7) Multiplying the transfer function by $C'(s)$, we get $C'_m(s) = \frac{1}{s+1}\,C'(s) = -\frac{1}{s+1}\cdot\frac{0.5}{s^2}$
8) Inverse Laplace transforming: $C'_m(t) = \mathcal{L}^{-1}\{C'_m(s)\} = \mathcal{L}^{-1}\left\{\frac{-0.5}{s^2(s+1)}\right\} = -0.5\,\mathcal{L}^{-1}\left\{\frac{1}{s^2(s+1)}\right\}$

   Eq. 2.28 “Table of Laplace Transforms” in “Reference Tables for students”

2.28	$\frac{\alpha}{s^2(s+\alpha)}$	$t-\frac{1}{\alpha}[1-\exp(-\alpha t)]$

$$C'_m(t) = -0.5\{t-[1-\exp(-t)]\} =$$
$$= \frac{1}{2} - \frac{\exp(-t)}{2} - \frac{t}{2}$$

9) The warning light turns on when $C_m(t) < 1$.
   Remember $C_m = C'_m + C_{m_{ss}}$.

$$C_m = 2 + \left(\frac{1}{2} - \frac{\exp(-t)}{2} - \frac{t}{2}\right)$$

   We can solve for the time t such that $C_m(t) = 1$, which gives t = 2.9 min ≈3min.
   Using the Hints (without a numerical solver) we can proceed by trial and error:
   t=2 min

$$C_m = 2 + \left(\frac{1}{2} - \frac{\exp(-2)}{2} - \frac{2}{2}\right) = 2 + \left(\frac{1}{2} - \frac{0.14}{2} - 1\right) \sim 1.4$$

   t=3 min

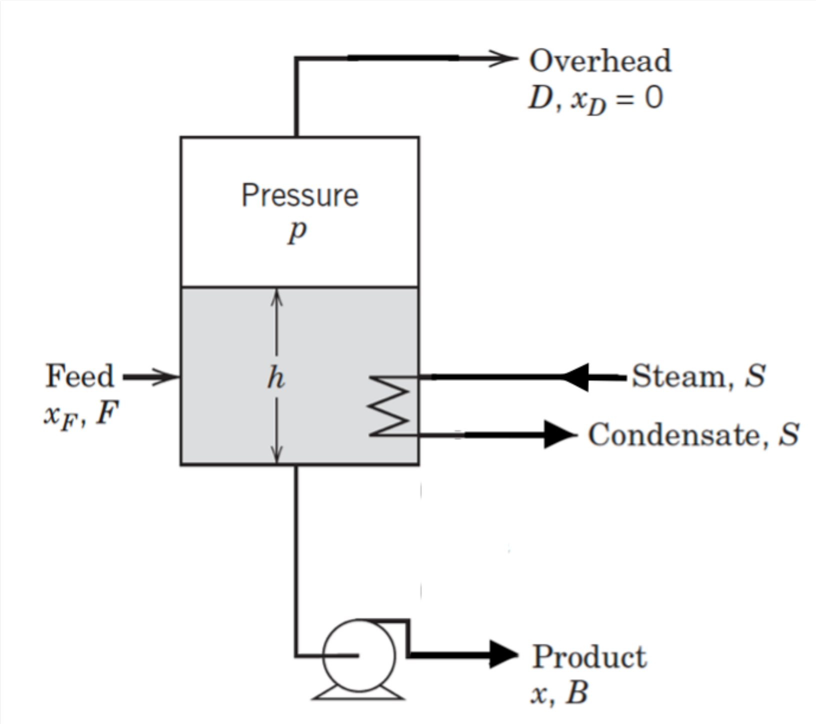
$$C_m = 2 + \left(\frac{1}{2} - \frac{\exp(-3)}{2} - \frac{3}{2}\right) = 2 + \left(\frac{1}{2} - \frac{0.05}{2} - 1.5\right) \sim 0.975$$

   At t=3 min the value of C_m is lower than 1, the warning light will turn on.

Section 4: CONTROL AND MONITORING

4.1. The feedback control

A feedback control is to be performed of a continuous evaporator (see the figure) that concentrates a product from the mass fraction x_F in the feed to the larger one x_B in the bottom stream.
The manipulated variable is the flow rate of saturated steam S.

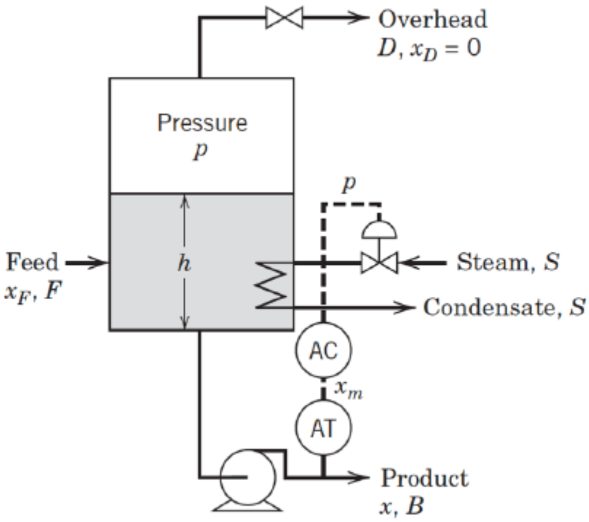


1. propose, on the same drawing, **the P&ID**
2. select the **controlled variable**
3. select the **disturbance variable** (if any)
4. draw the **block diagram for process control**

Among the various process **block components** (tank, valves, pump, etc.) individuate on the P&ID the characteristic variables of automatic control present in this process:

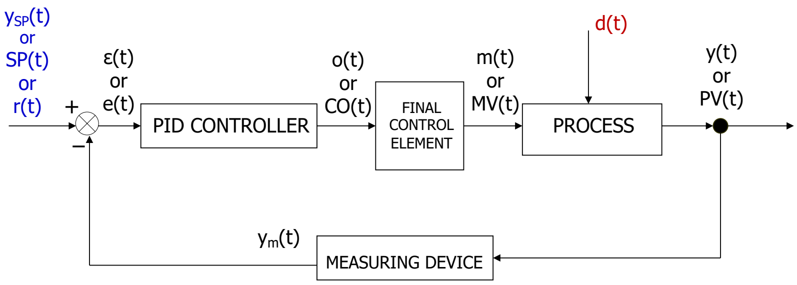
5. select the **sensor/measuring device**
6. select the **comparator**
7. select the **actuator**
8. select the **final control element**
9. what type of signal is used in the **control loop?**
10. what is the role of the tank in the **control loop system?**

1. A possible P&ID to feedback control the outlet composition is the following:



(Note that in this problem only the "manipulated variable" was assigned: flow rate of saturated steam S. Therefore, feedback control in which the measured and controlled variable is different than composition (i.e. level) would be equally right.)

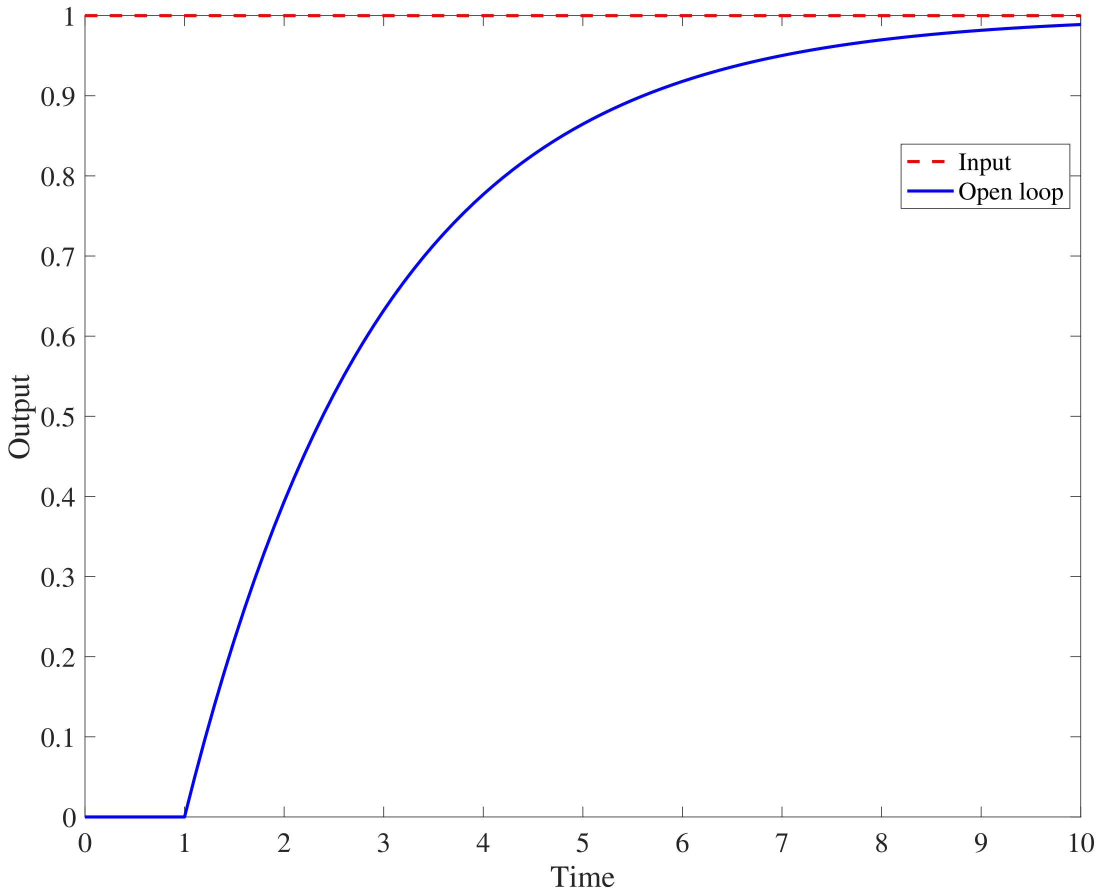
2. Outlet stream composition;
3. Feed flow rate and composition;
4.



5. AT
6. AC
7. Valve actuator
8. Valve
9. Electrical
10. Process

Section 5: CONTROLLERS

An unknown process at open loop is stimulated, at time 0 s, in its input by a step function (red dashed line in the attached figure) and, in the open loop configuration, the response of the process variable (to be controlled at closed loop) is recorded (blue solid line in the attached figure).



1. From the dynamic response determine the value of the **dead time** t_d

$$t_d = 1$$

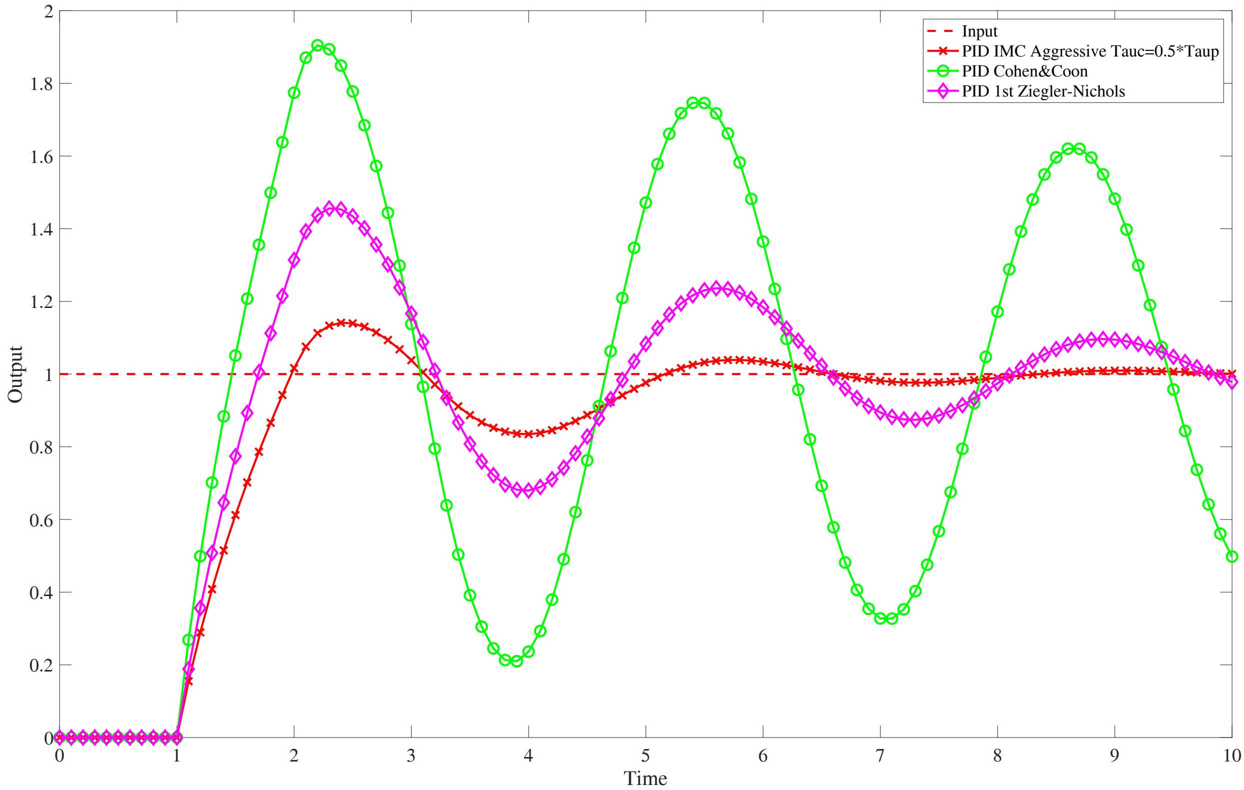
2. Obtain the transfer function of an FOPDT fitting model for which you may take K_p=1 and τ_p=2 s;

$$G(s) = \frac{K_p}{\tau_p\, s + 1} \exp(-t_d\, s) = \frac{1}{2\, s + 1} \exp\,(-s)$$

3. Describe **in a few words** the meaning of the FOPDT model

A true process model is usually neither first-order nor linear. Only the simplest processes exhibit such ideal dynamics. Therefore, in order to account for higher-order dynamics that are neglected in a first-order model, a time-delay term can be included in a first-order model. This modification can improve the agreement between model and experimental responses.

Different open loop tuning methods are used to find the optimal PID controller parameters and the resulting closed loop system responses are recorded and reported in the next attached figure.



4. Which is the best, according to the provided figure, tuning algorithm? (A qualitatively but detailed answer is required)

The IMC aggressive algorithm outperforms the Cohen and Coon and the 1st Ziegler-Nichols methods in terms of both speed and accuracy. While the latter two methods result in high overshoot and fail to reach the new steady state value within the recorded time range, the IMC aggressive algorithm is able to quickly bring the system to the closed loop configuration at the new steady state value.

5. According to the best tuning algorithm proposed by you in 4), calculate the tuning parameters of the PID controller.

From the IMC tuning tables ("IMC_tuning_tables.pdf" or "PID_formule_tuning.pdf"

	K_C	τ_I	τ_D
PID Ideal	$\frac{1}{K_P}\left(\frac{\tau_P+0.5\theta_P}{\tau_C+0.5\theta_P}\right)$	$\tau_P+0.5\theta_P$	$\frac{\tau_P\theta_P}{2\tau_P+\theta_P}$

From the legend of the graph: $\tau_C = 0.5\,\tau_P$

Therefore:

τ_C=1.00

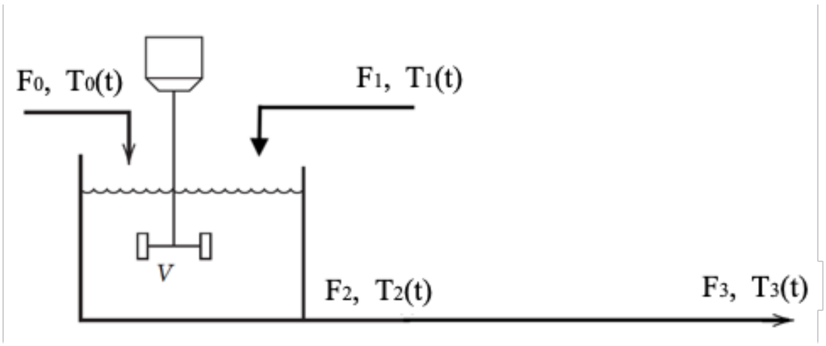
$K_c = 1.67$

τ_I =2.50

$\tau_D = 0.40$

Section 6: MATHEMATICAL MODELLING OF A LUMPED PARAMETER SYSTEM

A perfectly stirred, constant-volume tank has two input streams, both consisting of the same liquid. The temperature of both the inlet streams can vary with time, whereas the volumetric flow rates F_0, F_1 and F_2 are constant. The thermal-physical properties can be assumed constant, and the output line is relatively short, so that a negligible time delay for this line can be expected $\left(T_3(t) = T_2(t)\right)$.



You must

1. write the **dynamical model** of the system;
2. write the **steady state** model of the system;
3. list **input**, **state**, **output** variables and the **parameters** of the model;
4. is the dynamical model a linear model? If not, **individuate and indicate the non-linear terms**.
5. **write the model in the Laplace domain;**
6. **obtain the transfer functions** describing the relation between the input and output variables;
7. **classify the obtained transfer functions** and individuate the parameters.

A maintenance intervention on the plant modifies the path of the output line, increasing its length. Therefore, the assumption of negligible time delay t_d for the output line does not hold anymore and it is $\left(T_3(t) \neq T_2(t)\right)$.

In such a new situation you must:

8. Write the **dynamical model** of the system;
9. **write the model in the Laplace domain**;
10. **obtain the transfer functions** describing the relation between the input and output variables.

1. ACC=IN-OUT+~~GEN~~

$$\rho V C_p \frac{dT_2(t)}{dt} = \rho F_0 C_p\left(T_0(t) - T_{ref}\right) + \rho F_1 C_p\left(T_1(t) - T_{ref}\right) - \rho F_2 C_p\left(T_2(t) - T_{ref}\right)$$

2.

$$0 = \rho F_0 C_p\left(T_{0_{ss}} - T_{ref}\right) + \rho F_1 C_p\left(T_{1_{ss}} - T_{ref}\right) - \rho F_2 C_p\left(T_{2_{ss}} - T_{ref}\right)$$

3. Input: $T_0(t), T_1(t)$
   State: $T_2(t)$
   Output: $T_2(t)$
   Parameters: $\rho, V, C_p, F_0, F_1, F_2$

4. The model is linear.
5. To rewrite the model in Laplace domain, the model in deviation variables has to be obtained:

$$\begin{aligned}\cancel{\rho} V \cancel{C_p} \frac{dT_2(t)}{dt} - 0 &= \left[\cancel{\rho} F_0 \cancel{C_p}\left(T_0(t) - T_{ref}\right) - \cancel{\rho} F_0 \cancel{C_p}\left(T_{0_{ss}} - T_{ref}\right)\right] \\ &+ \left[\cancel{\rho} F_1 \cancel{C_p}\left(T_1(t) - T_{ref}\right) - \cancel{\rho} F_1 \cancel{C_p}\left(T_{1_{ss}} - T_{ref}\right)\right] \\ &- \left[\cancel{\rho} F_2 \cancel{C_p}\left(T_2(t) - T_{ref}\right) - \cancel{\rho} F_2 \cancel{C_p}\left(T_{2_{ss}} - T_{ref}\right)\right]\end{aligned}$$

$$V\frac{dT_2'(t)}{dt} = F_0 T_0'(t) + F_1 T_1'(t) - F_2 T_2'(t)$$

$$\mathcal{L}\left\{V\frac{dT_2'(t)}{dt} + F_2 T_2'(t) = F_0 T_0'(t) + F_1 T_1'(t)\right\}$$

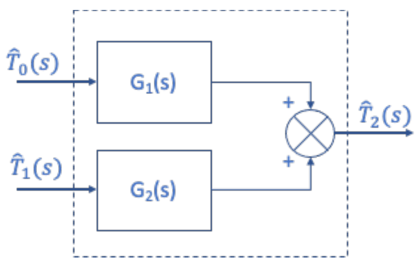
$$V\hat{T}_2 s + F_2\hat{T}_2 = F_0\hat{T}_0 + F_1\hat{T}_1$$

6.

$$\hat{T}_2(V\,s + F_2) = F_0\hat{T}_0 + F_1\hat{T}_1$$

$$\hat{T}_2 = \frac{F_0}{V\,s + F_2}\hat{T}_0 + \frac{F_1}{V\,s + F_2}\hat{T}_1$$

$$\hat{T}_2 = \frac{\frac{F_0}{F_2}}{\frac{V}{F_2}s + 1}\hat{T}_0 + \frac{\frac{F_1}{F_2}}{\frac{V}{F_2}s + 1}\hat{T}_1$$



$$G_1(s) = \frac{\hat{T}_2}{\hat{T}_0} = \frac{\frac{F_0}{F_2}}{\frac{V}{F_2}s + 1} = \frac{K_{P_1}}{\tau\,s + 1}$$

$$G_2(s) = \frac{\hat{T}_2}{\hat{T}_1} = \frac{\frac{F_1}{F_2}}{\frac{V}{F_2}s + 1} = \frac{K_{P_2}}{\tau\, s + 1}$$

7. Both $G_1(s)$ and $G_2(s)$ are first order transfer function. The parameters are $K_{P_1} = F_0/F_2$, $K_{P_2} = F_1/F_2$, $\tau = V/F_2$.

8. Energy balance on the tank and relationship between delayed variables (note that $F_3 = F_2 = constant$)):

$$\begin{cases} \rho V C_p \dfrac{dT_2(t)}{dt} = \rho F_0 C_p \left(T_0(t) - T_{ref}\right) + \rho F_1 C_p \left(T_1(t) - T_{ref}\right) - \rho F_3 C_p \left(T_2(t) - T_{ref}\right) \\ T_3(t) = T_2(t - t_D) \end{cases}$$

9. $$\mathcal{L}\begin{cases} \rho V C_p \frac{dT_2(t)}{dt} = \rho F_0 C_p \left(T_0(t) - T_{ref}\right) + \rho F_1 C_p \left(T_1(t) - T_{ref}\right) - \rho F_3 C_p \left(T_2(t) - T_{ref}\right) \\ T_3(t) = T_2(t - t_D) \end{cases}$$

$$\begin{cases} \hat{T}_2 = \dfrac{K_{P_1}}{\tau\, s + 1}\hat{T}_0 + \dfrac{K_{P_2}}{\tau\, s + 1}\hat{T}_1 \\ \hat{T}_3 = \hat{T}_2 \exp(-\mathrm{t_D}\, s) \end{cases}$$

10.

$$\hat{T}_3 = \frac{K_{P_1}\exp(-\mathrm{t_D}\, s)}{\tau\, s + 1}\hat{T}_0 + \frac{K_{P_2}\exp(-\mathrm{t_D}\, s)}{\tau\, s + 1}\hat{T}_1$$

$$G_1(s) = \frac{\hat{T}_3}{\hat{T}_0} = \frac{K_{P_1}}{\tau\, s + 1}\exp(-\mathrm{t_D}\, s)$$

$$G_2(s) = \frac{\hat{T}_3}{\hat{T}_1} = \frac{K_{P_2}}{\tau\, s + 1}\exp(-\mathrm{t_D}\, s)$$