Surname **Name** **Student's code.:**

There is no need
Indeed, it is NOT allowed to use a programmable calculator!

Section 1: TRUE/FALSE QUIZZES

1. As General Guideline for Specifying the Controller Action (Direct or Reverse): the overall product of the gains for all of the components in the feedback control loop must be positive.

   true ☐ false ☐

2. In a transfer function G(s), the variable s belongs to the set of complex numbers.

   true ☐ false ☐

3. A linear dynamical system is BIBO marginally stable if its transfer function has a pole with a null real part with multiplicity higher than one.

   true ☐ false ☐

4. In a regulator problem the aim is to track the time varying set-point

   true ☐ false ☐

Section 2: QUIZZES

1. Which of the following ones is not an **actuator**?

a. ☐ hydraulic ram
b. ☐ centrifugal pump
c. ☐ *relay* controller
d. ☐ heating or cooling element

2. The Laplace transform cannot be used to solve

a. ☐ second order differential equations
b. ☐ linear or linearised differential equations
c. ☐ nonlinear differential equations
d. ☐ higher-order linear differential equations

5. Which of the following "parameters" is not included in the 2nd order system law?

a. ☐ Process gain
b. ☐ Dead time
c. ☐ Natural oscillation period
d. ☐ Damping factor

Section 3: REFERENCE DYNAMIC MODELS

3.1. Response of a dynamic model

A chemical reaction is taking place in a tank, and the concentration of a reactant is being monitored by a concentration analyzer. The relationship between the measured concentration $C'_m(s)$ and the actual concentration $C'(s)$ is given by the following transfer function (in deviation variable form):

$$\frac{C'_m(s)}{C'(s)} = \frac{1}{s+1}$$

The system is at its steady-state (SS) value, with actual and measured concentration of 2 mol/L: C_{ss}= $C_{m_{ss}}$= 2 mol/L. A warning light on the analyzer turns on whenever the measured concentration drops below 1.5 mol/L or above 3 mol/L. Suppose that at time t = 0 min, the concentration of the reactant in the tank begins to vary exponentially, $C(t) \; = 2 \exp\left(\frac{t[min]}{10}\right)$, where C has units of mol/L and t has units of minutes.

1. Which type of reference dynamic model is represented by the above transfer function?
2. How much is the steady-state gain?
3. How much is the time constant?
4. Is the process affected by dead time? If so, how much is the time delay?
5. Write the forcing function $C(t)$ in terms of deviation variable(s) $C'(t)$ in the time domain.
6. Transport the forcing function $C'(t)$ in the Laplace domain ($\hat{C}(s)$).
7. Obtain the expression, in the Laplace domain, of the measured concentration $\hat{C}_m(s)$.
8. Obtain the expression of the time evolution of the measured concentration in terms of deviation variable C'_m(t)
9. At what time, with the approximation of ±0.5 min, will the warning light turn on?

Hints:

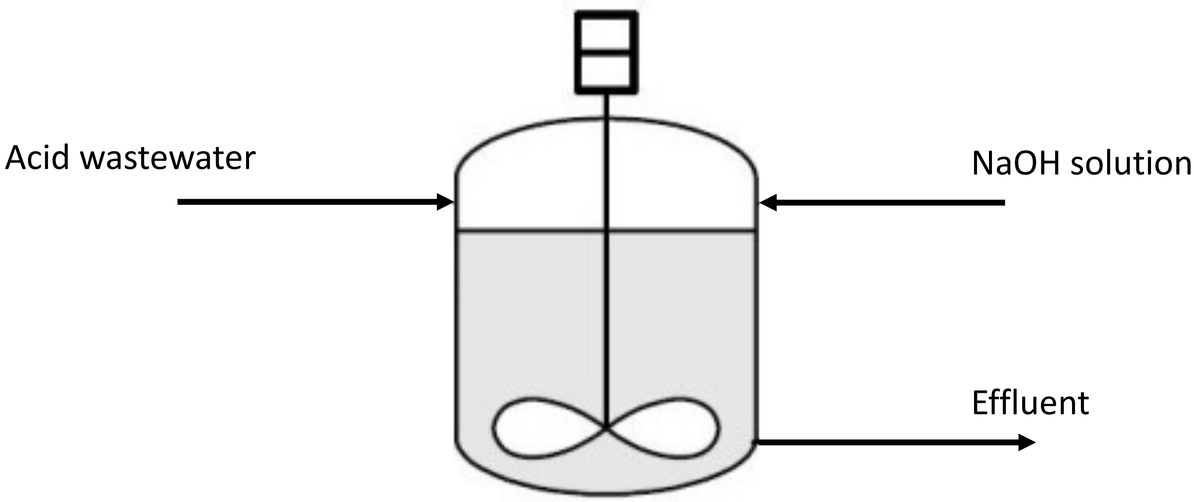
- The following table can be used to approximate the exponential decay function

t	*exp(-t)*	*exp(t/10)*
0	1.00	1.00
0.25	0.78	1.03
0.5	0.61	1.05
0.75	0.47	1.08
1	0.37	1.11
1.5	0.22	1.16
2	0.14	1.22
2.5	0.08	1.28
3	0.05	1.35
3.5	0.03	1.42
4	0.02	1.49
4.5	0.01	1.57

Section 4: CONTROL AND MONITORING

4.1. The feedback control

A feedback control has to be performed on a CSTR (see the figure) used in a neutralization process. The base (Sodium Hydroxide, NaOH) is added to bring up the pH level and neutralize the acid wastewater in order to obtain a well-defined pH of the effluent (**controlled variable**).
The NaOH solution flow rate is **the manipulated variable** of the feedback control loop**.**



1. propose, on the same drawing, **the P&ID**
2. select the **disturbance variable/s** (if any)
3. draw the **closed loop block diagram** for this particular process control

Among the various process **block components** (tank, valves, motor, etc.) individuate on the P&ID (sketched as an answer to the above question 1.) the characteristic **components** of automatic control present in this process:

4. select the **sensor/measuring device**
5. select the **comparator**
6. select the **actuator**
7. select the **final control element**
8. what type of signal is used in the **control loop?**
9. what is the role of the tank in the **control loop system?**

Section 5: CONTROLLERS

5.1 Closed-loop step response and controller type

The following figures show the **closed loop** response (y(t)) of the process to a unit step change of the **set point (y_sp(t)** in **orange color**) at t=0 s, along with the controller output (CO(t) in **blu dashed color**).

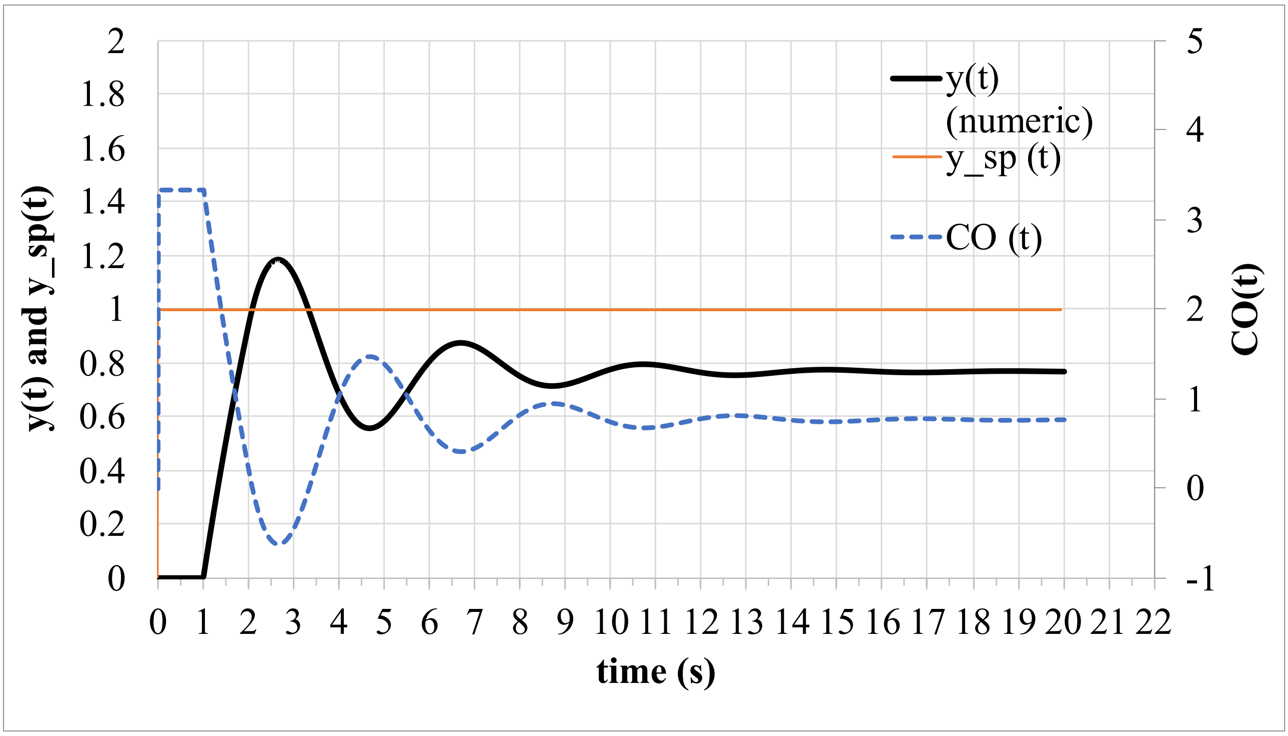


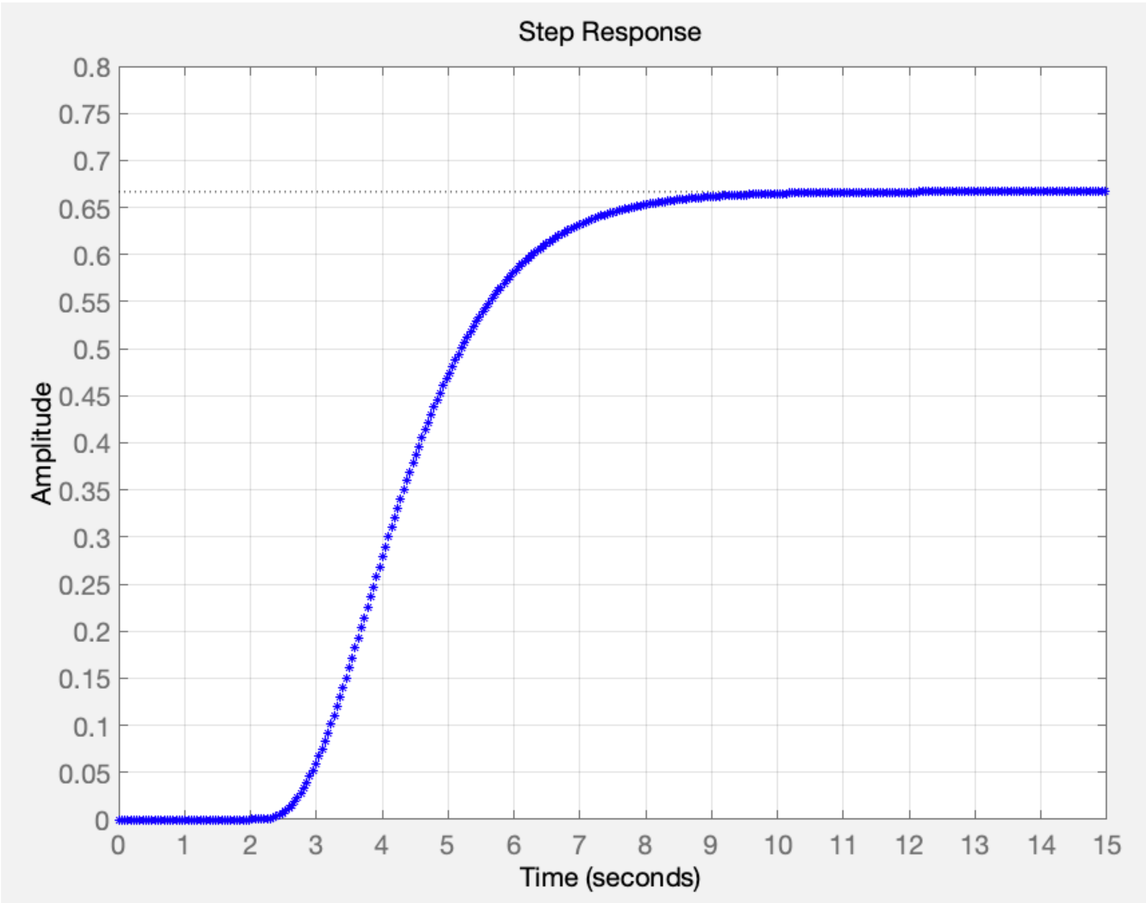
Figure 1. Controller type I

1. Is the process to be controller affected by time-delay?

2. Describe the type of controller in Figure 1 and describe its working principles, advantages, and disadvantages

5.2 Open-loop PID Tuning with the "process reaction curve" method of Cohen and Coon

An unknown process at open loop is stimulated, at time 0 s, in its input by a **unit step function** and, in the open loop configuration, the response of the process variable (to be controlled at closed loop) is recorded (blue solid stars in the attached figure).



Approximate the open loop transfer function, whose step response is in the figure above, with a FOPDT transfer function.

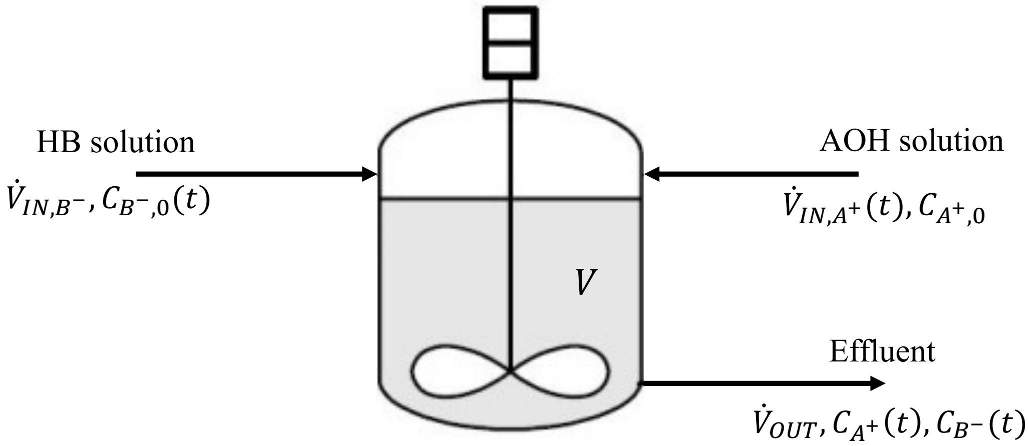
1. Determine the process gain Kp
2. Determine the time constant τ_P
3. Determine the dead time t_d
4. Which type of **PID controller** tuning formula can be used?

Section 6: MATHEMATICAL MODELLING OF A LUMPED PARAMETER SYSTEM

A CSTR (see the figure) is used in a neutralization process. The base is added to bring up the pH level and neutralize the acid wastewater to obtain a well-defined pH of the effluent.

The acid wastewater has a constant and high flow rate, whereas its pH can vary in time.

The base solution is prepared at constant and high concentration and can be dosed by varying its flow rate.



The neutralization reaction is the following:

$$AOH + HB \to A^+ + B^- + H_2O$$

$$(i.e. NaOH + HCl \to Na^+ + Cl^- + H_2O)$$

Since the neutralization between strong acids and bases, in perfectly mixed reactors, is instantaneous ($H^+ + OH^- \to H_2O$), the process can be modeled by looking at the moles of ions A^+ (i.e. Na^+) and B^- (i.e. Cl^-), **using as state variables the concentration of A^+ and B^- (C_{A^+} and C_{B^-}) and not considering any reaction kinetics.**[1]

The following simplifying assumptions are valid:

- $C_{A^+,0}$ **is constant**;
- $\dot{V}_{IN,B^-}$ **is constant**;
- $C_{A^+,0} \gg C_{B^+,0}(t)$, then the volumetric flow rate of the base is negligible with respect to the volumetric flow rate of the acid wastewater solution ($\dot{V}_{IN,A}(t) \ll \dot{V}_{IN,B}$), which means that $\dot{V}_{OUT} \sim \dot{V}_{IN,B}$ **is constant** and that the reaction volume **V is constant** as well.

[1] **Not to be used in this exam:** once the ions concentration is known, the H^+ (and OH^-) concentration can be calculated using the electroneutrality and the water dissociation equations.

You must:

1. Write the **dynamical model** of the system;
2. write the **steady state** model of the system;
3. list **input**, **state**, **output** variables and the **parameters** of the model;
4. is the dynamical model a linear model? If not, **individuate and indicate the non-linear terms**.
5. **write the model in the Laplace domain;**
6. **obtain the transfer functions** describing the relation between the input and output variables;
7. **classify the obtained transfer functions** and individuate the parameters.

If the assumption of $C_{A0} \gg C_{B0}(t)$ **was not valid anymore** (and also $\dot{V}_{IN,A}(t) \ll \dot{V}_{IN,B}$), but the assumption of **constant volume of reaction was still valid**:

8. would the dynamical model still be solvable with the Laplace transform? If so, how?