
Solving linear programming problems using the graphical method

Example - designing a diet

A dietitian wants to design a breakfast menu for certain hospital patients. The menu is to include two items **A** and **B**. Suppose that each ounce of **A** provides 2 units of vitamin C and 2 units of iron and each ounce of **B** provides 1 unit of vitamin C and 2 units of iron. Suppose the cost of **A** is 4¢/ounce and the cost of **B** is 3¢/ounce. If the breakfast menu must provide at least 8 units of vitamin C and 10 units of iron, how many ounces of each item should be provided in order to meet the iron and vitamin C requirements for the least cost? What will this breakfast cost?

x = #oz. of **A**
y = #oz. of **B**

vit. C: $2x + y \geq 8$
iron: $2x + 2y \geq 10$
$x \geq 0, y \geq 0$

Cost = $C = 4x+3y$

x = #oz. of **A**

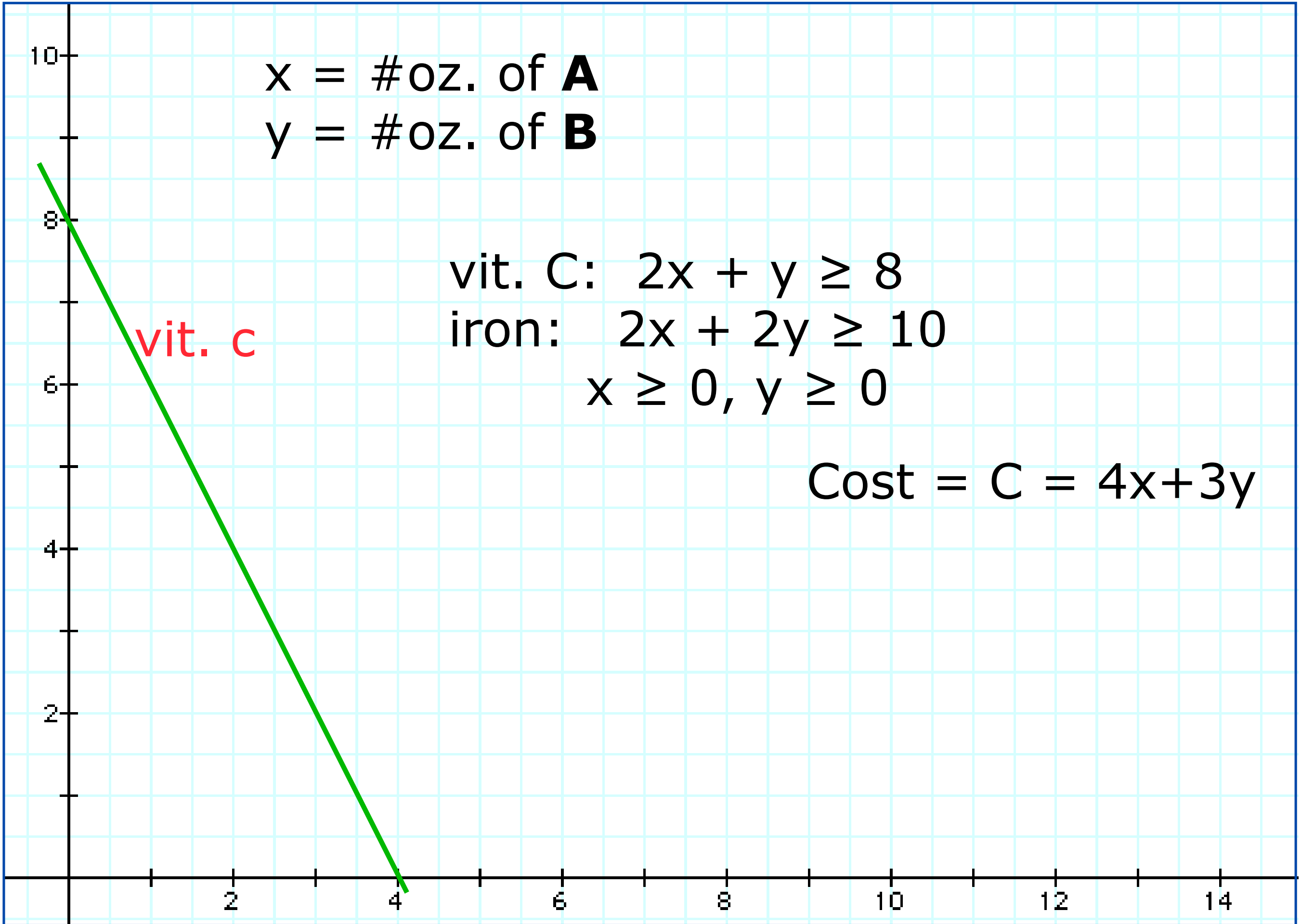
y = #oz. of **B**

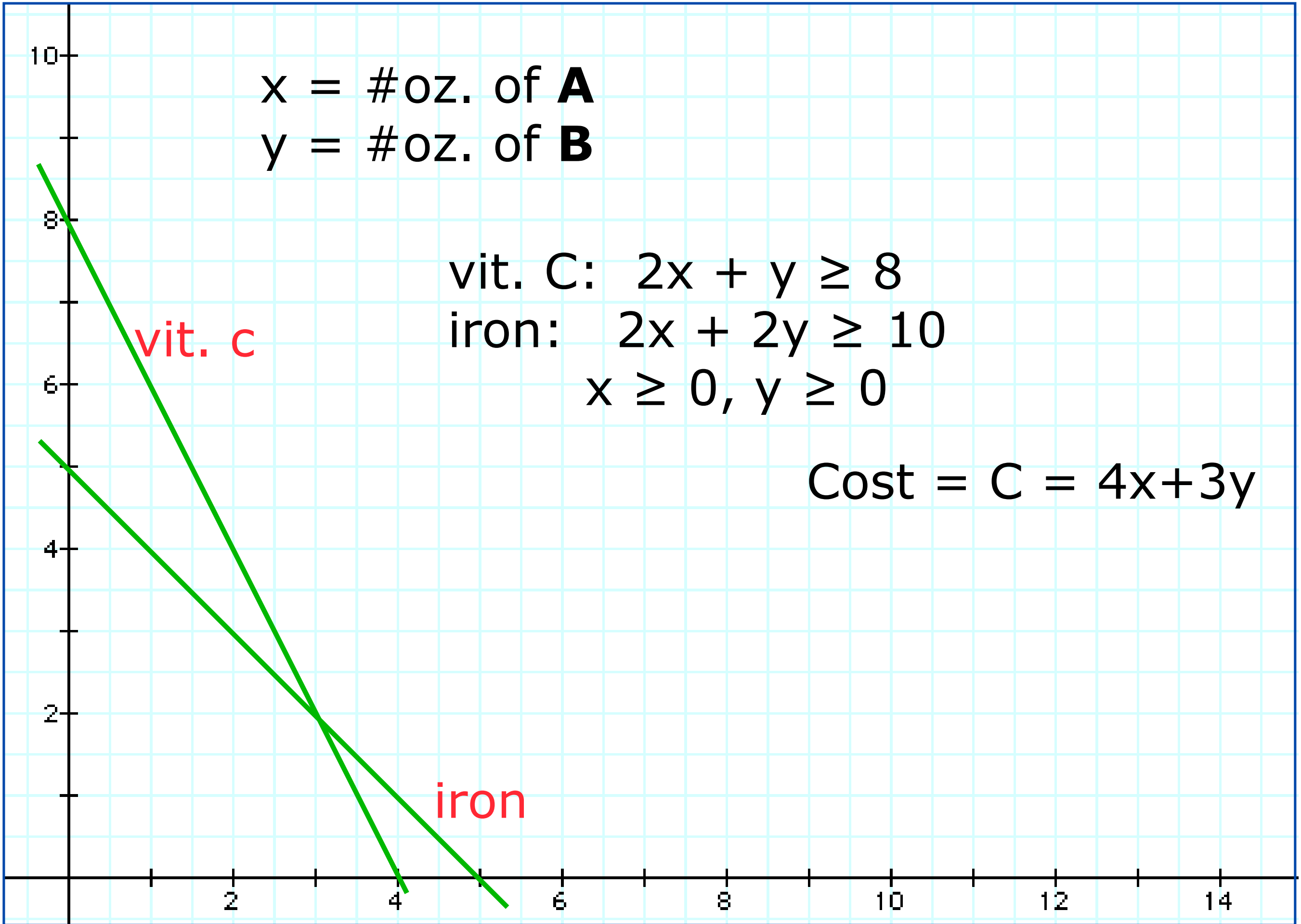
vit. C: $2x + y \geq 8$

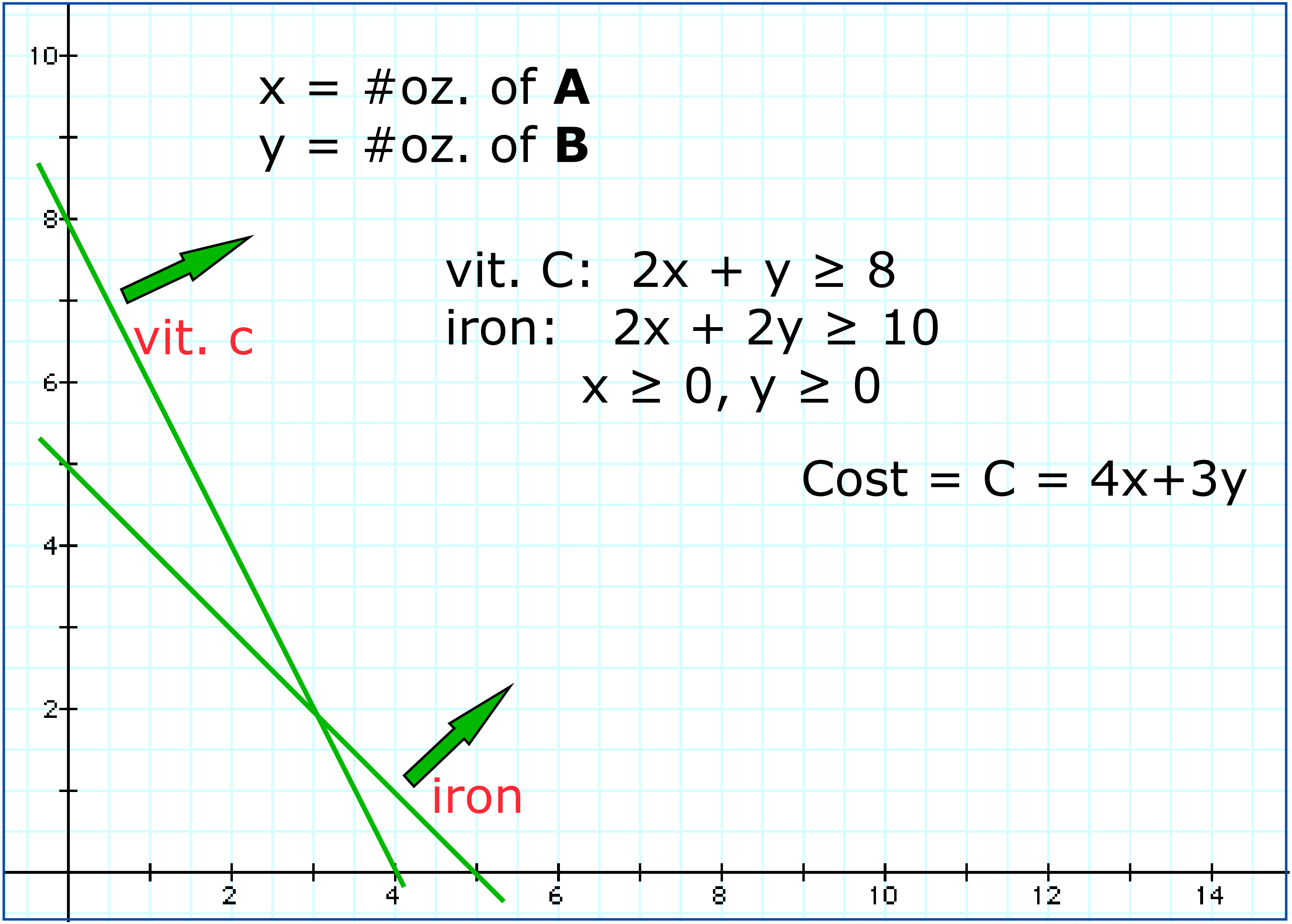
iron: $2x + 2y \geq 10$

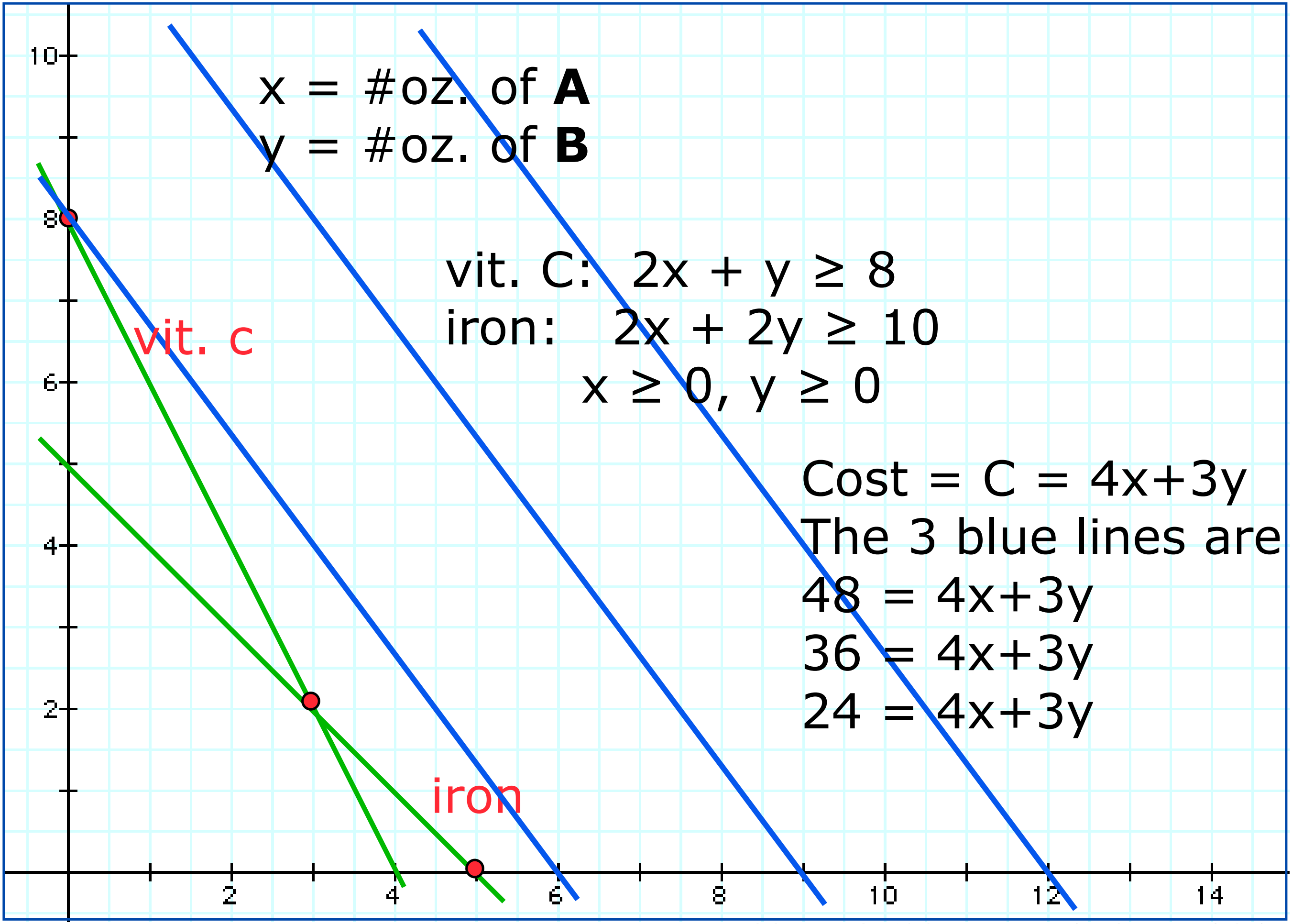
$x \geq 0, y \geq 0$

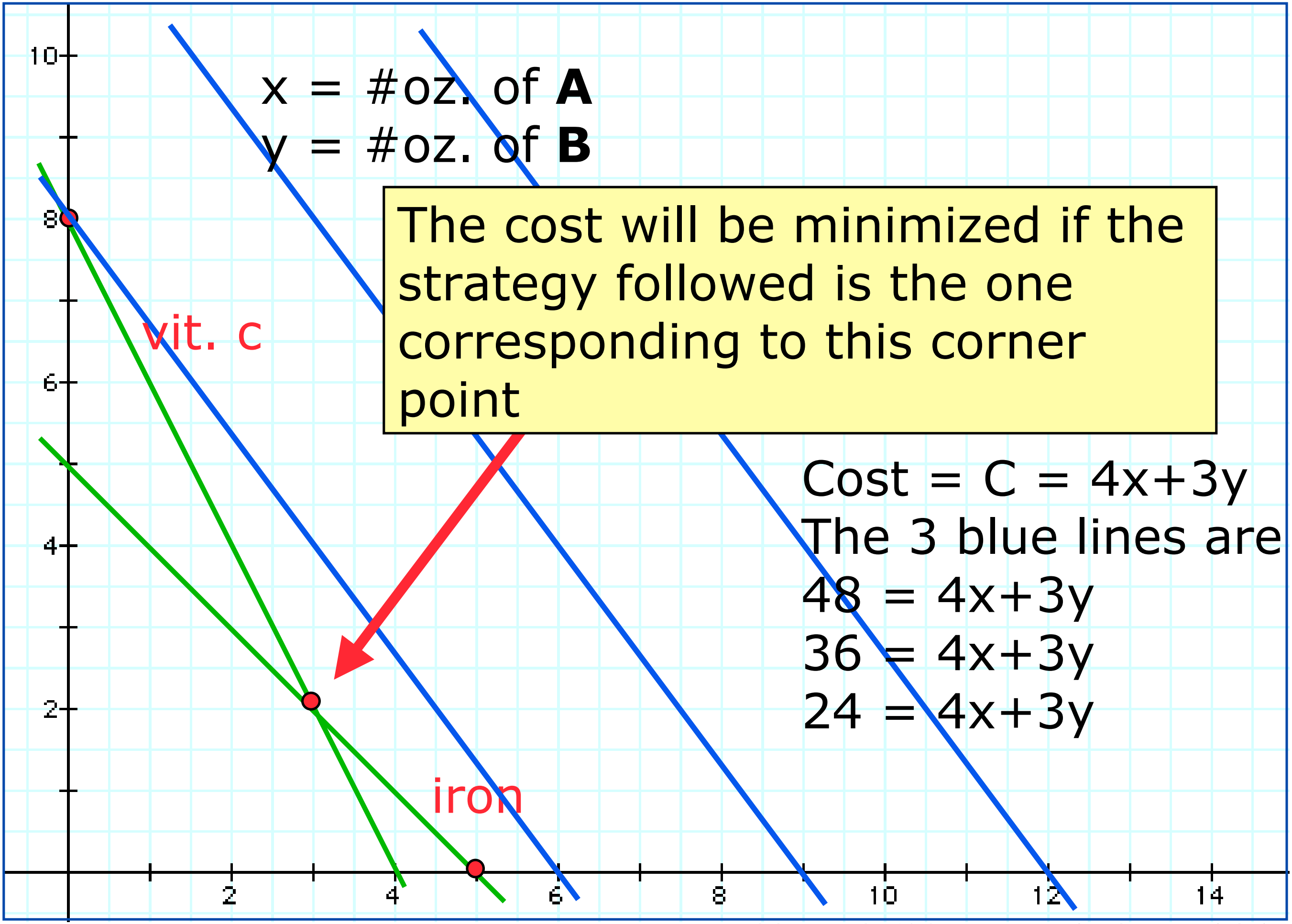
Cost = $C = 4x+3y$

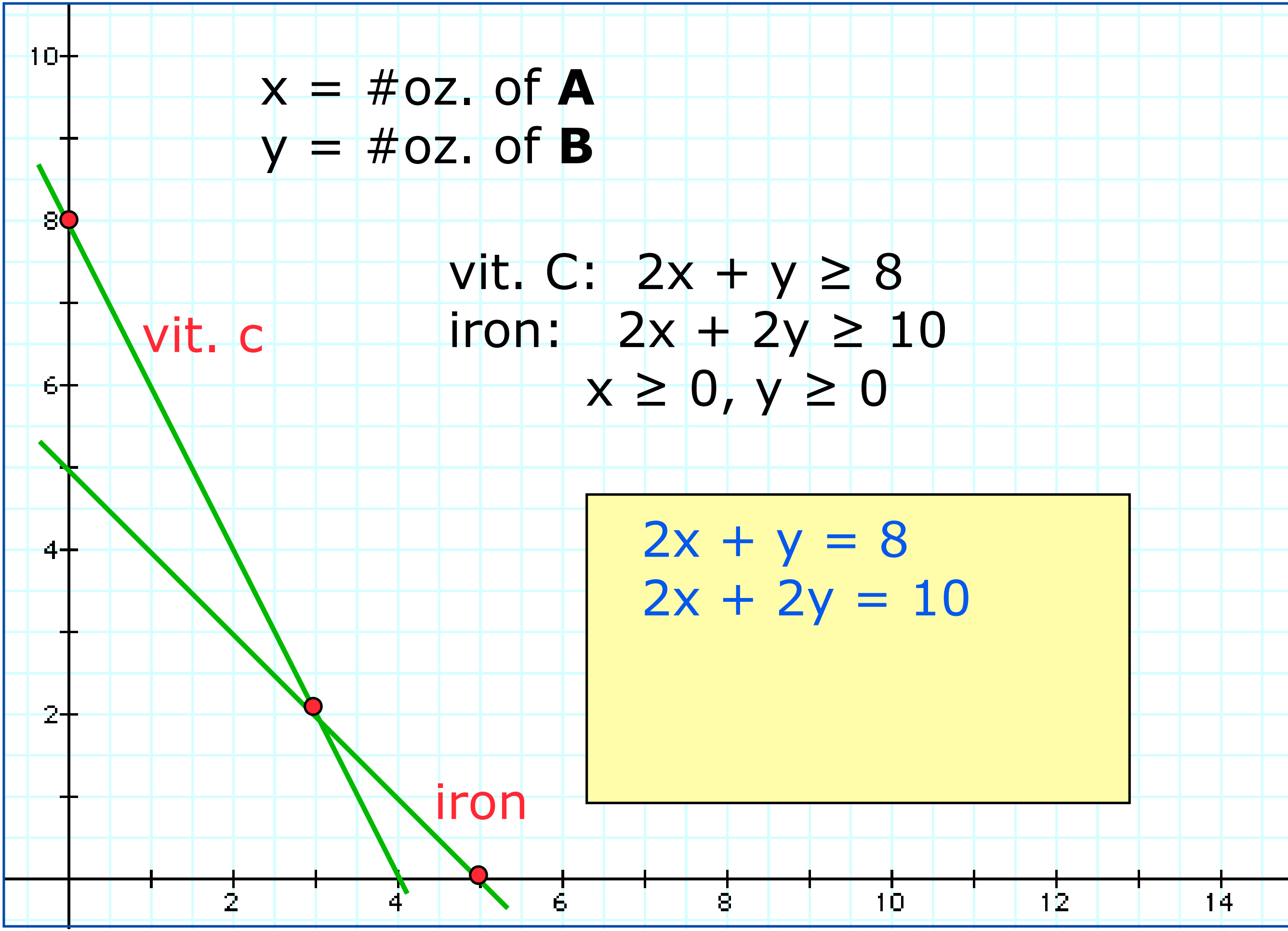


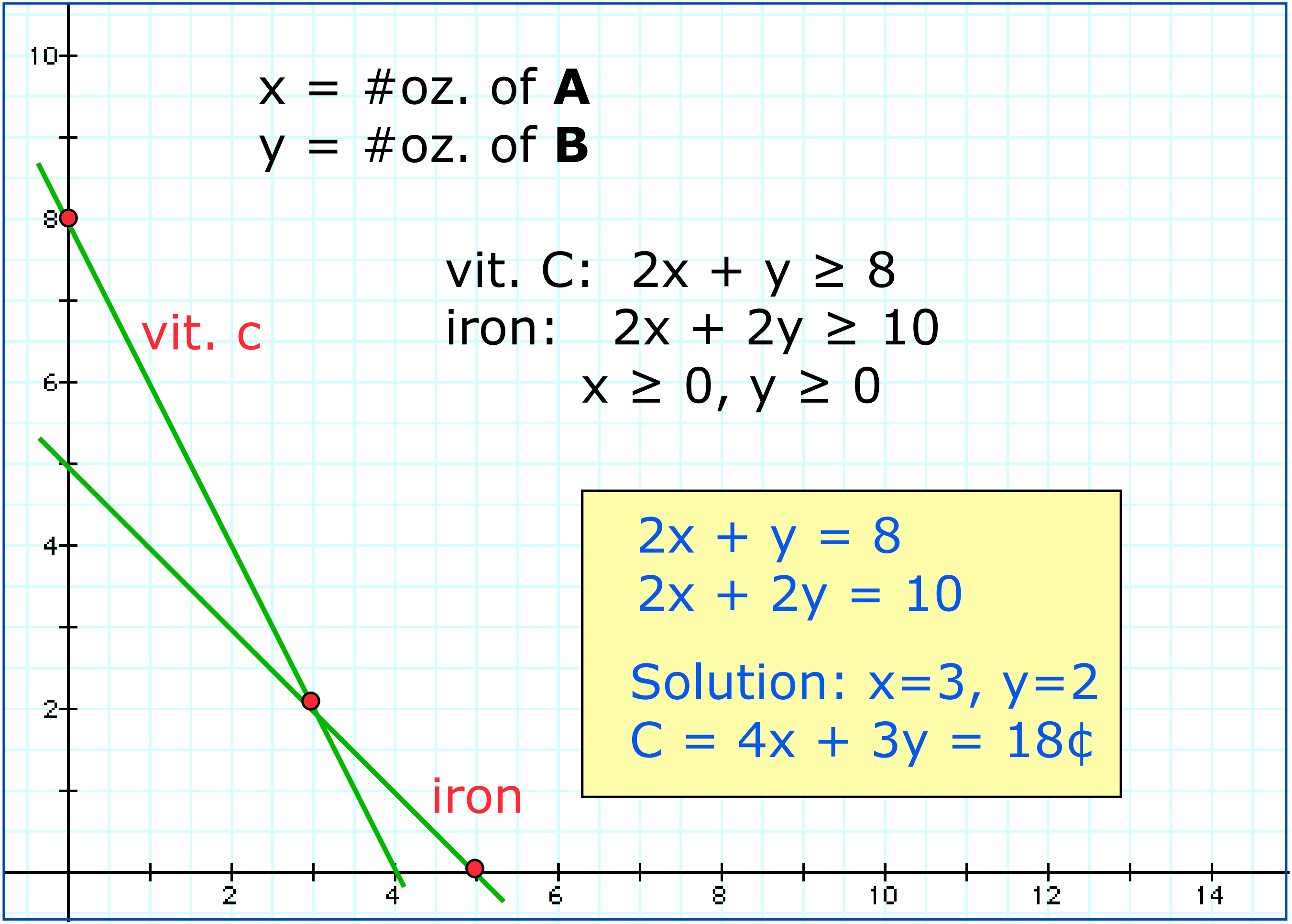

x = #oz. of **A**
y = #oz. of **B**
vit. C: $2x + y \geq 8$
iron: $2x + 2y \geq 10$
$x \geq 0, y \geq 0$
Cost = C = $4x+3y$
vit. c
iron
10
8
6
4
2
2
4
6
8
10
12
14

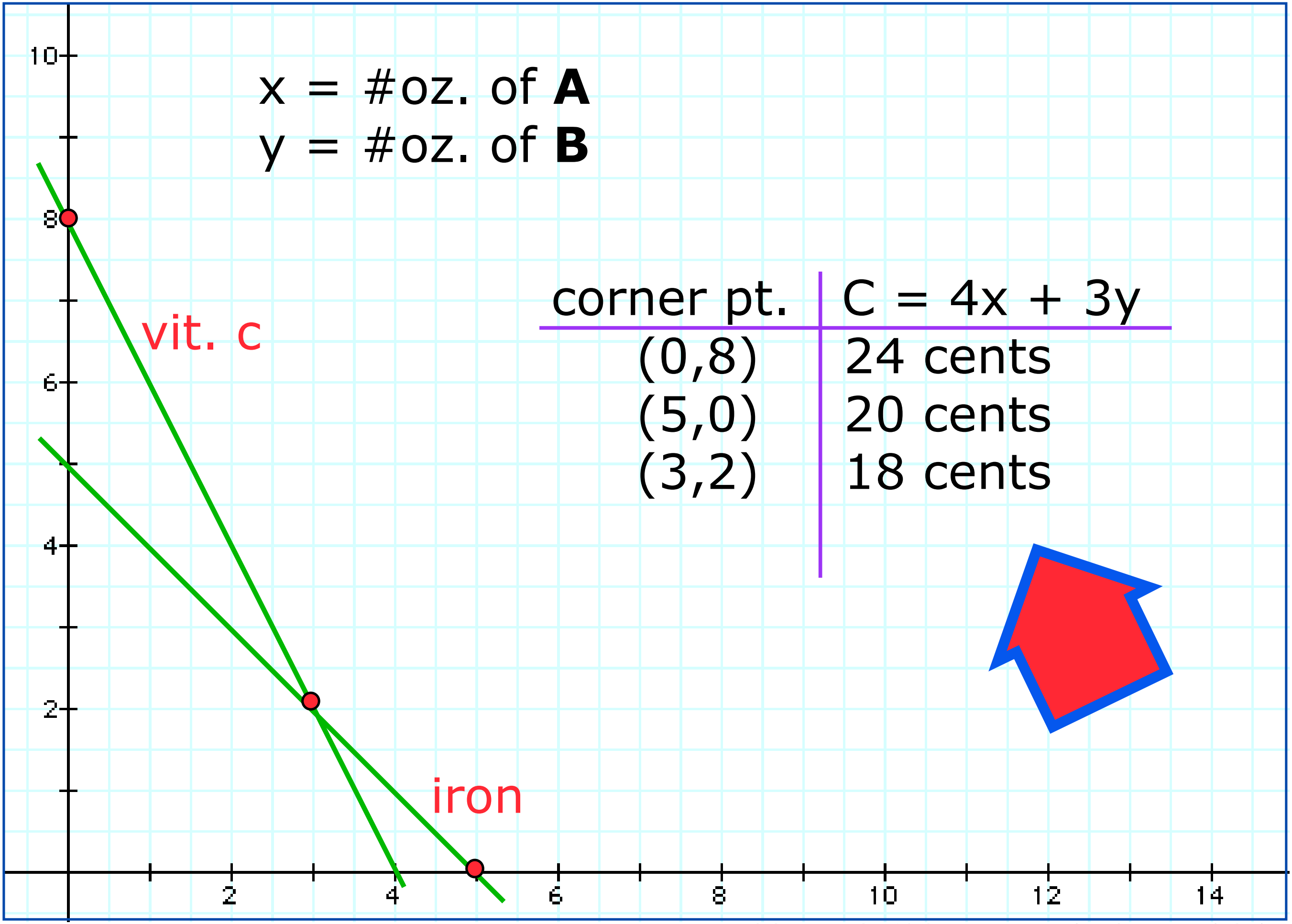

x = #oz. of **A**
y = #oz. of **B**
vit. C: 2x + y ≥ 8
iron: 2x + 2y ≥ 10
x ≥ 0, y ≥ 0
Cost = C = 4x+3y
vit. c
iron
10
8
6
4
2
2
4
6
8
10
12
14


x = #oz. of **A**
y = #oz. of **B**
vit. C: 2x + y ≥ 8
iron: 2x + 2y ≥ 10
x ≥ 0, y ≥ 0
Cost = C = 4x+3y
The 3 blue lines are
48 = 4x+3y
36 = 4x+3y
24 = 4x+3y
vit. c
iron
10
8
6
4
2
2
4
6
8
10
12
14


x = #oz. of **A**
y = #oz. of **B**
The cost will be minimized if the strategy followed is the one corresponding to this corner point
vit. c
iron
Cost = C = 4x+3y
The 3 blue lines are
48 = 4x+3y
36 = 4x+3y
24 = 4x+3y
10
8
6
4
2
2
4
6
8
10
12
14


x = #oz. of **A**
y = #oz. of **B**
vit. C: 2x + y ≥ 8
iron: 2x + 2y ≥ 10
x ≥ 0, y ≥ 0
vit. c
iron
2x + y = 8
2x + 2y = 10
10
8
6
4
2
2
4
6
8
10
12
14


x = #oz. of **A**
y = #oz. of **B**
vit. C: 2x + y ≥ 8
iron: 2x + 2y ≥ 10
x ≥ 0, y ≥ 0
vit. c
iron
2x + y = 8
2x + 2y = 10
Solution: x=3, y=2
C = 4x + 3y = 18¢
10
8
6
4
2
2
4
6
8
10
12
14



corner pt.	C = 4x + 3y
(0,8)	24 cents
(5,0)	20 cents
(3,2)	18 cents

Example - bicycle factories

A small business makes 3-speed and 10-speed bicycles at two different factories. Factory **A** produces 16 3-speed and 20 10-speed bikes in one day while factory **B** produces 12 3-speed and 20 10-speed bikes daily. It costs $1000/day to operate factory **A** and $800/day to operate factory **B**. An order for 96 3-speed bikes and 140 10-speed bikes has just arrived. How many days should each factory be operated in order to fill this order at a minimum cost? What is the minimum cost?

x = # days factory **A** is operated
y = # days factory **B** is operated

x = # days factory **A** is operated
y = # days factory **B** is operated

x = # days factory **A** is operated
y = # days factory **B** is operated

3-speed constraint: $16x + 12y \geq 96$

x = # days factory **A** is operated

y = # days factory **B** is operated

3-speed constraint: $16x + 12y \geq 96$

10-speed constraint: $20x + 20y \geq 140$

$x \geq 0, y \geq 0$

x = # days factory **A** is operated

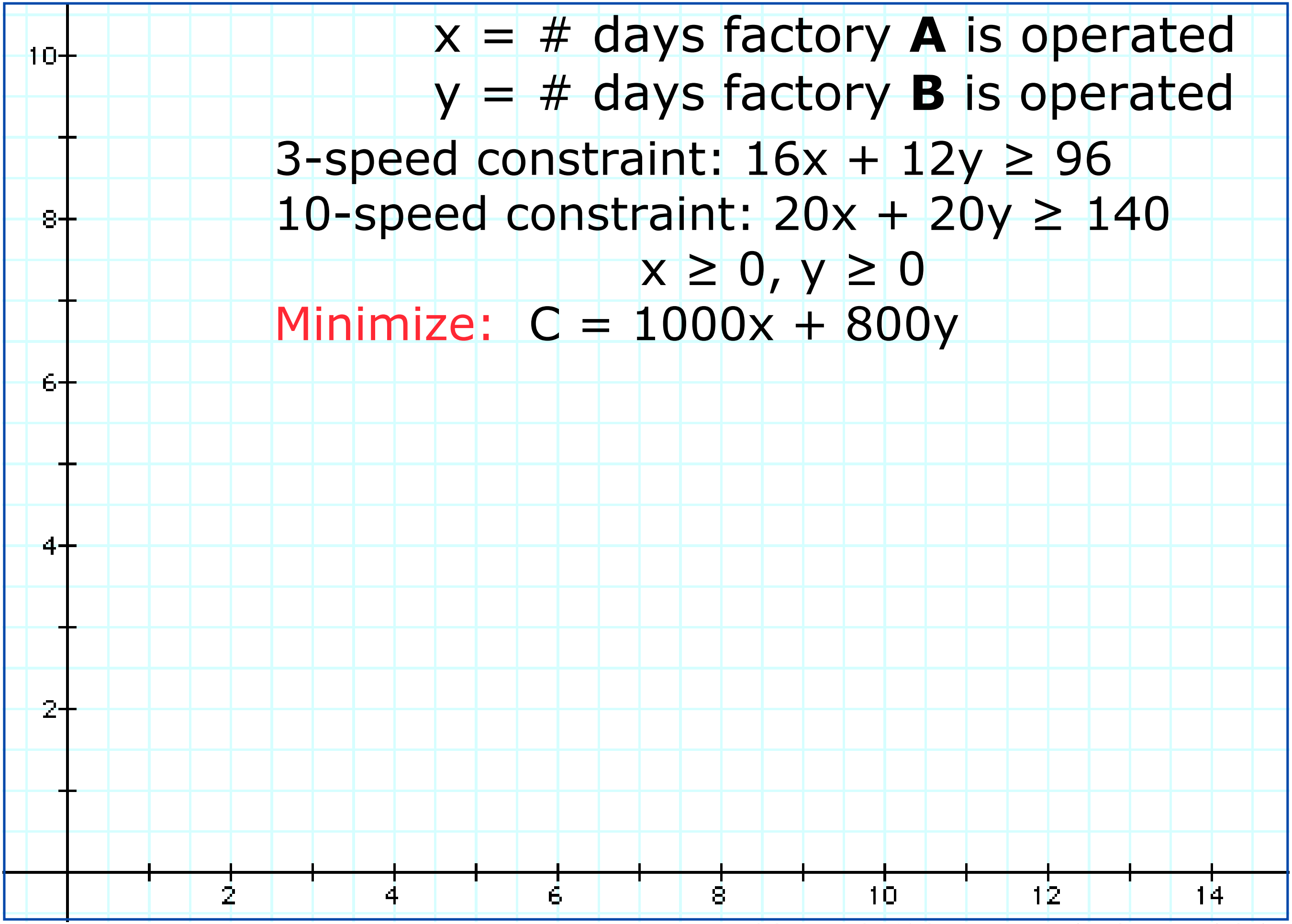
y = # days factory **B** is operated

3-speed constraint: $16x + 12y \geq 96$

10-speed constraint: $20x + 20y \geq 140$

$x \geq 0,\ y \geq 0$

Minimize: $C = 1000x + 800y$



x = # days factory **A** is operated

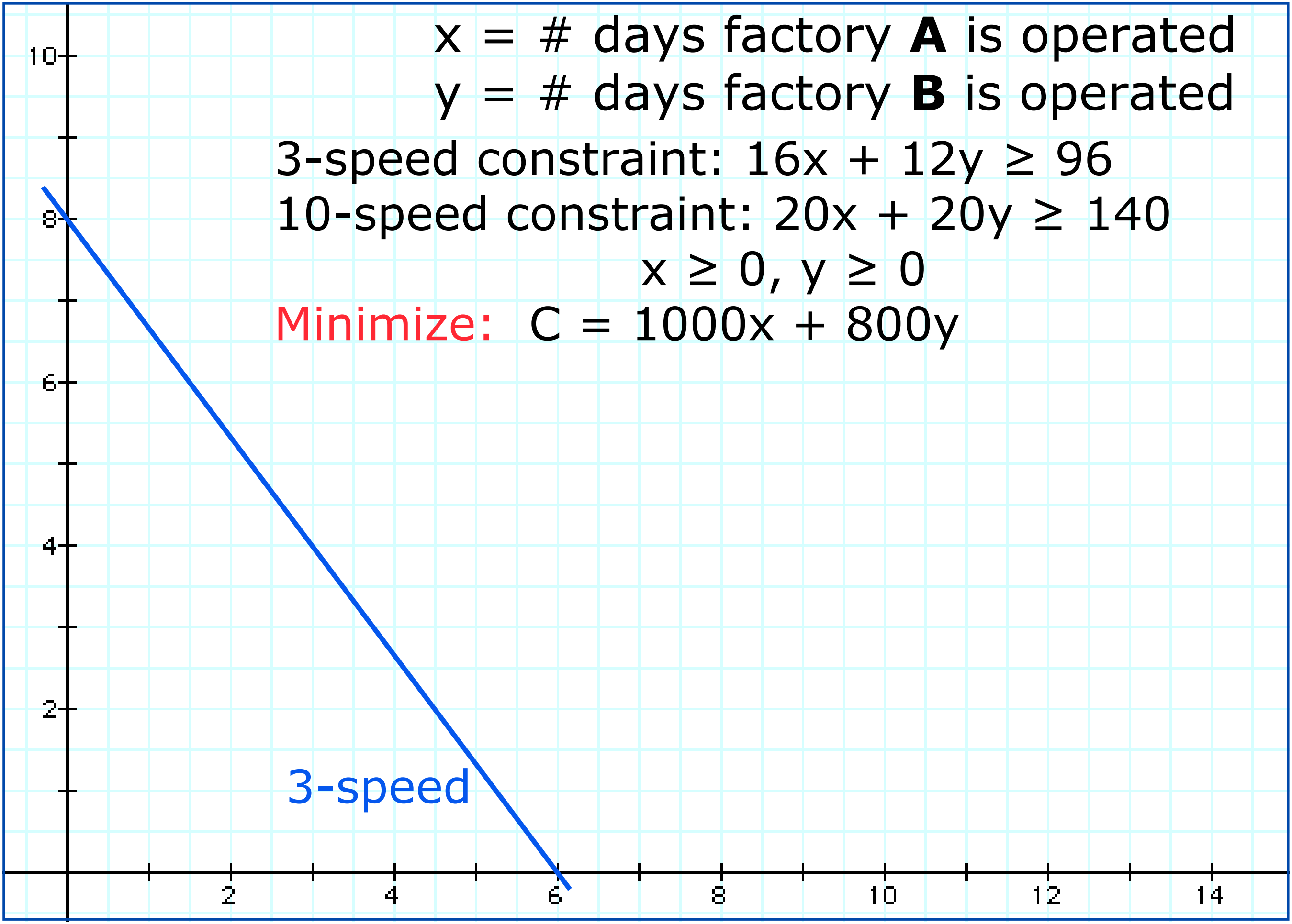
y = # days factory **B** is operated

3-speed constraint: $16x + 12y \geq 96$

10-speed constraint: $20x + 20y \geq 140$

$x \geq 0,\ y \geq 0$

Minimize: $C = 1000x + 800y$



x = # days factory **A** is operated

y = # days factory **B** is operated

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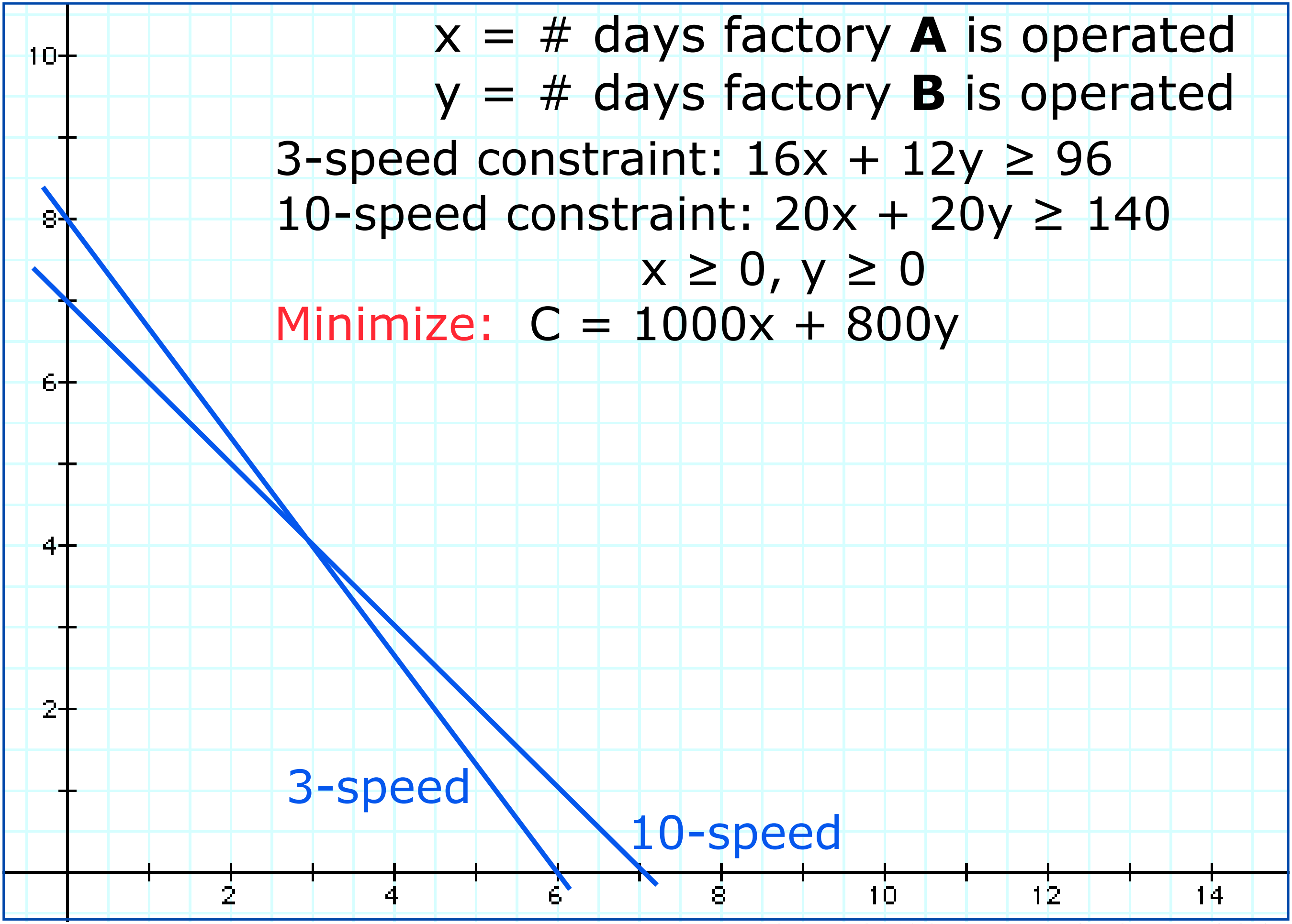
Minimize: $C = 1000x + 800y$



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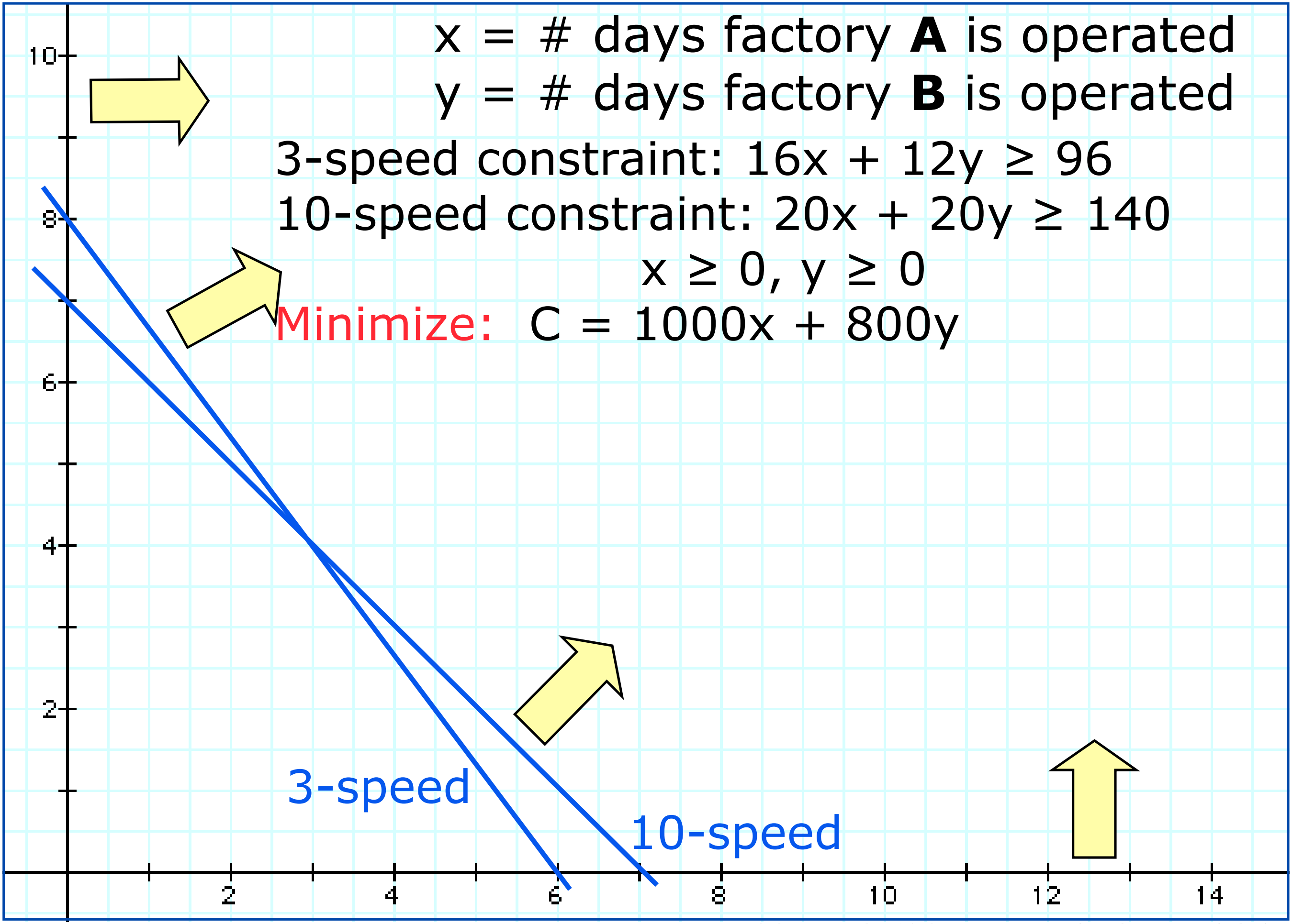
y = # days factory **B** is operated

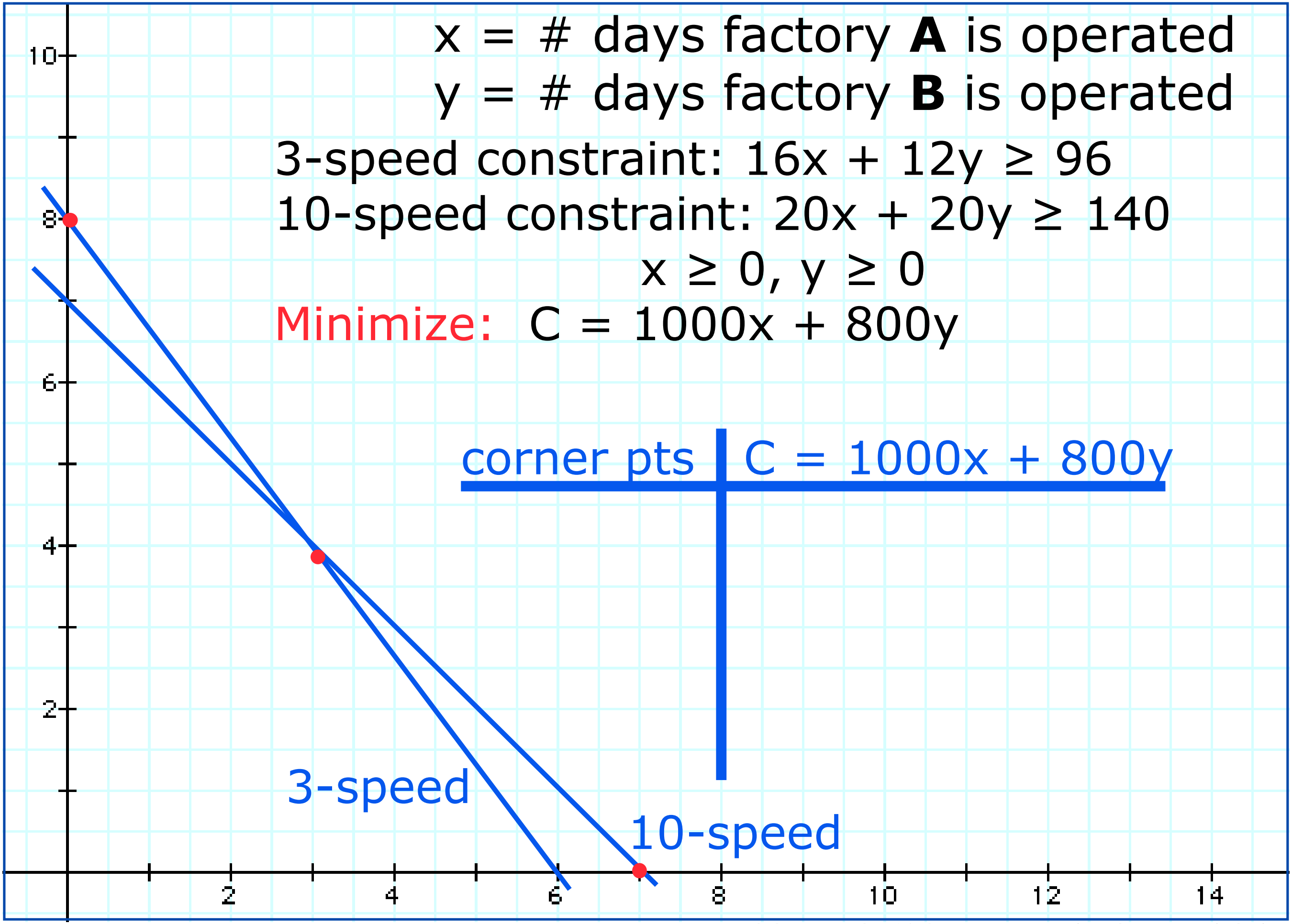
3-speed constraint: $16x + 12y \geq 96$

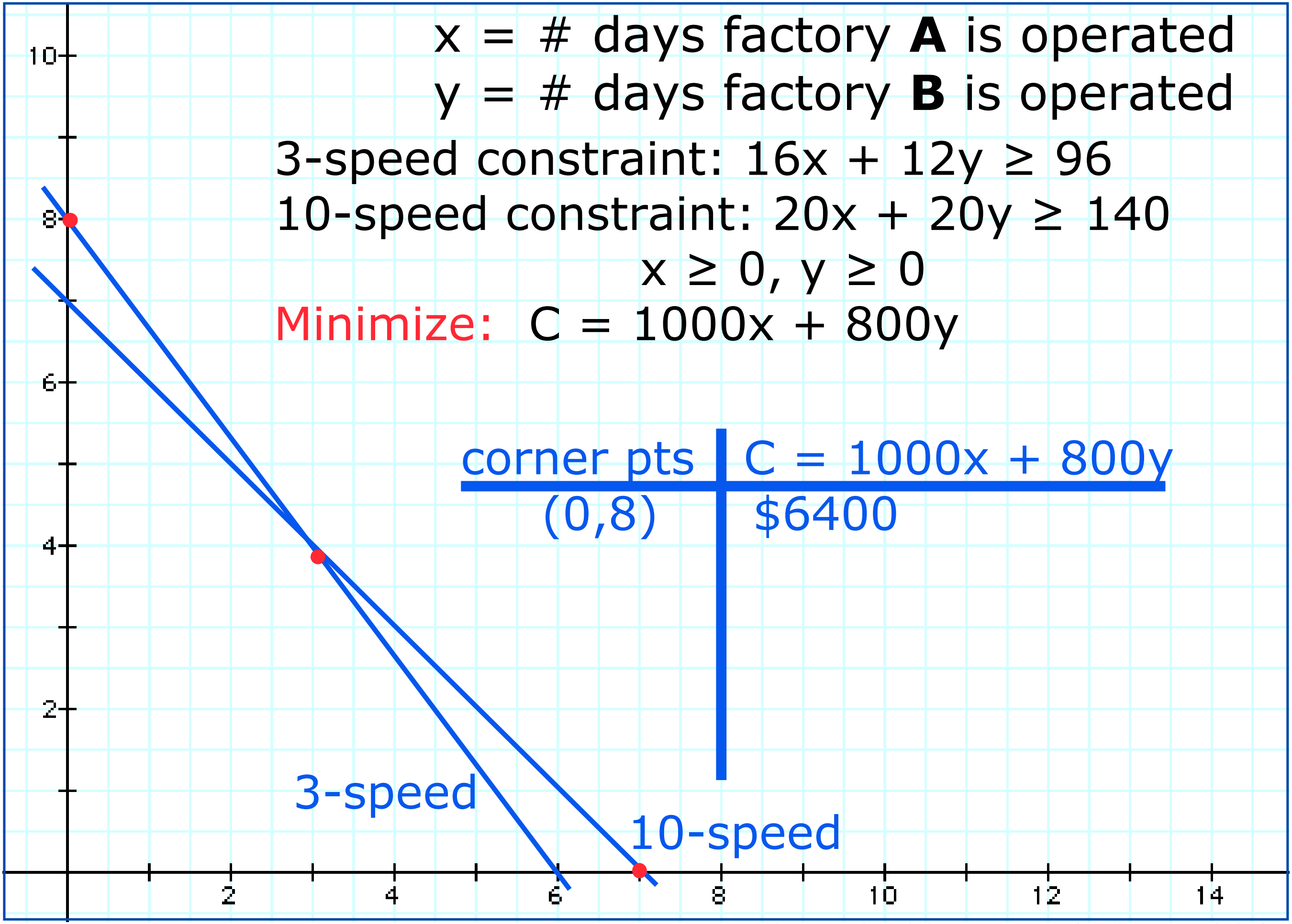
10-speed constraint: $20x + 20y \geq 140$

$x \geq 0, \; y \geq 0$

Minimize: $C = 1000x + 800y$




x = # days factory **A** is operated
y = # days factory **B** is operated
3-speed constraint: $16x + 12y \geq 96$
10-speed constraint: $20x + 20y \geq 140$
$x \geq 0, y \geq 0$
Minimize: $C = 1000x + 800y$
corner pts
C = 1000x + 800y
3-speed
10-speed
10
8
6
4
2
2
4
6
8
10
12
14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y = # days factory **B** is operated
3-speed constraint: $16x + 12y \geq 96$
10-speed constraint: $20x + 20y \geq 140$
$x \geq 0,\ y \geq 0$
Minimize: $C = 1000x + 800y$
corner pts | C = 1000x + 800y
(0,8) | $6400
3-speed
10-speed
10
8
6
4
2
2
4
6
8
10
12
14

x = # days factory **A** is operated

y = # days factory **B** is operated

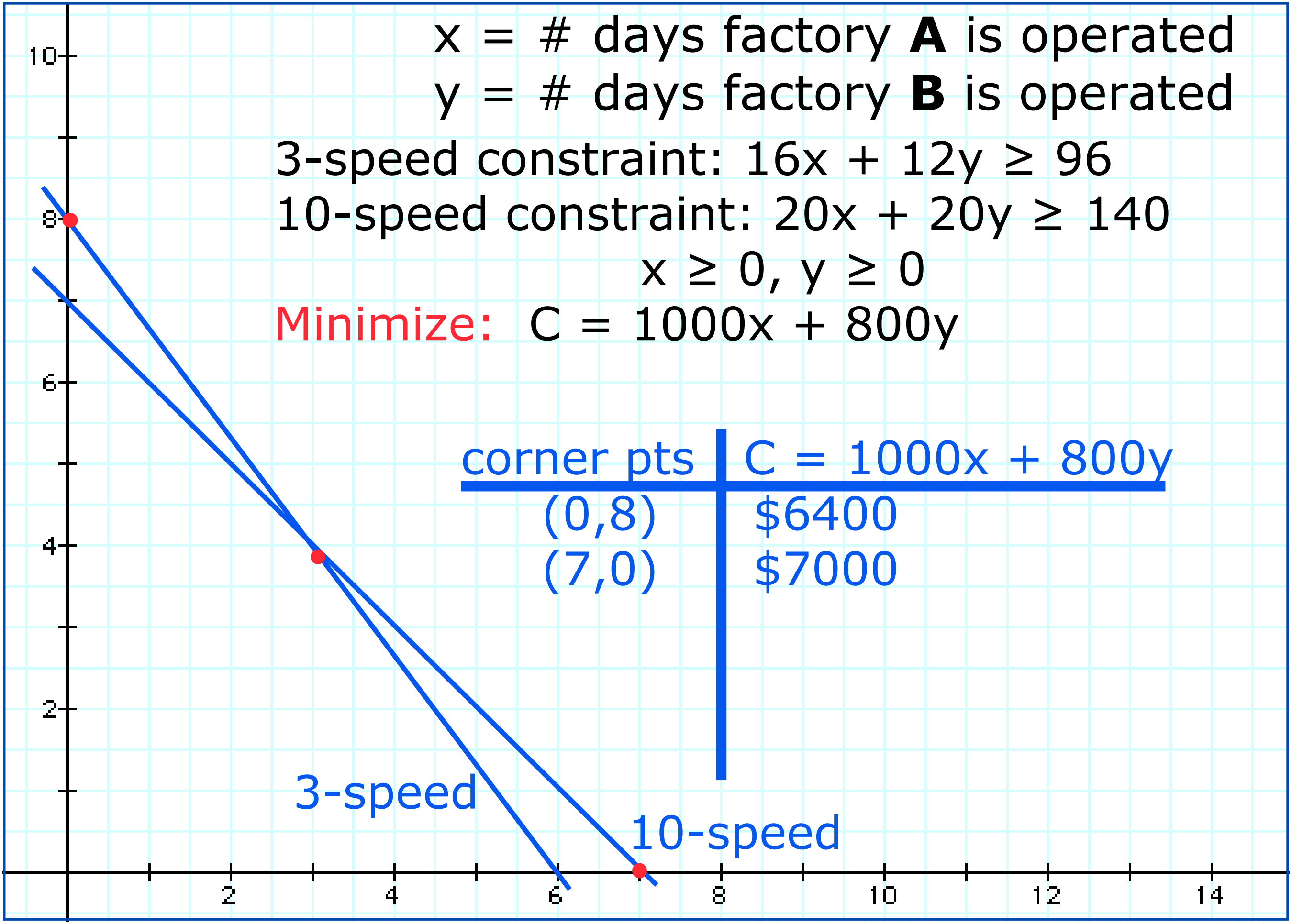
3-speed constraint: $16x + 12y \geq 96$

10-speed constraint: $20x + 20y \geq 140$

$x \geq 0,\ y \geq 0$

Minimize: $C = 1000x + 800y$

corner pts	$C = 1000x + 800y$
(0,8)	\$6400
(7,0)	\$7000



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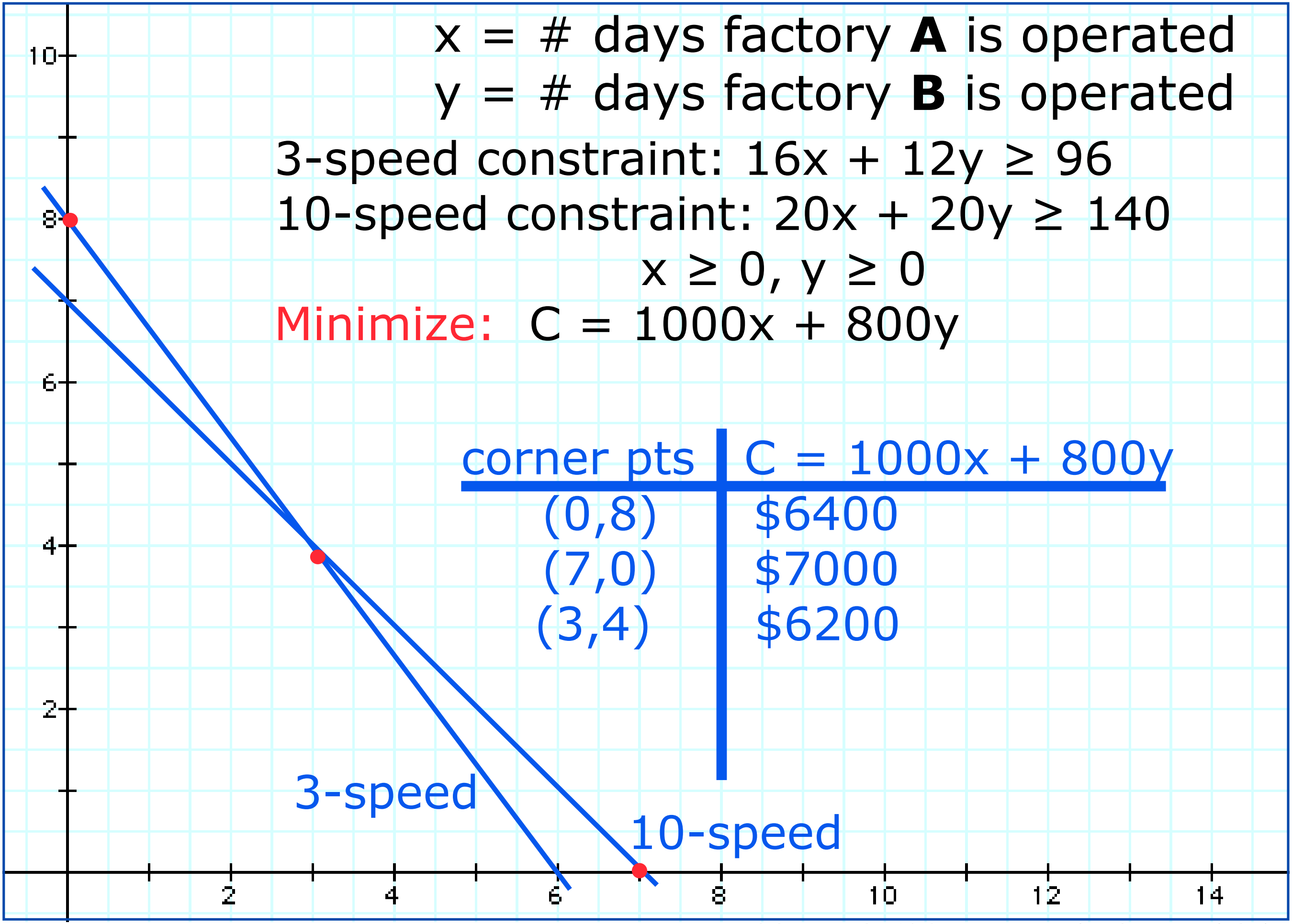
3-speed constraint: $16x + 12y \geq 96$

10-speed constraint: $20x + 20y \geq 140$

$x \geq 0,\ y \geq 0$

Minimize: $C = 1000x + 800y$

corner pts	C = 1000x + 800y
(0,8)	$6400
(7,0)	$7000
(3,4)	$6200



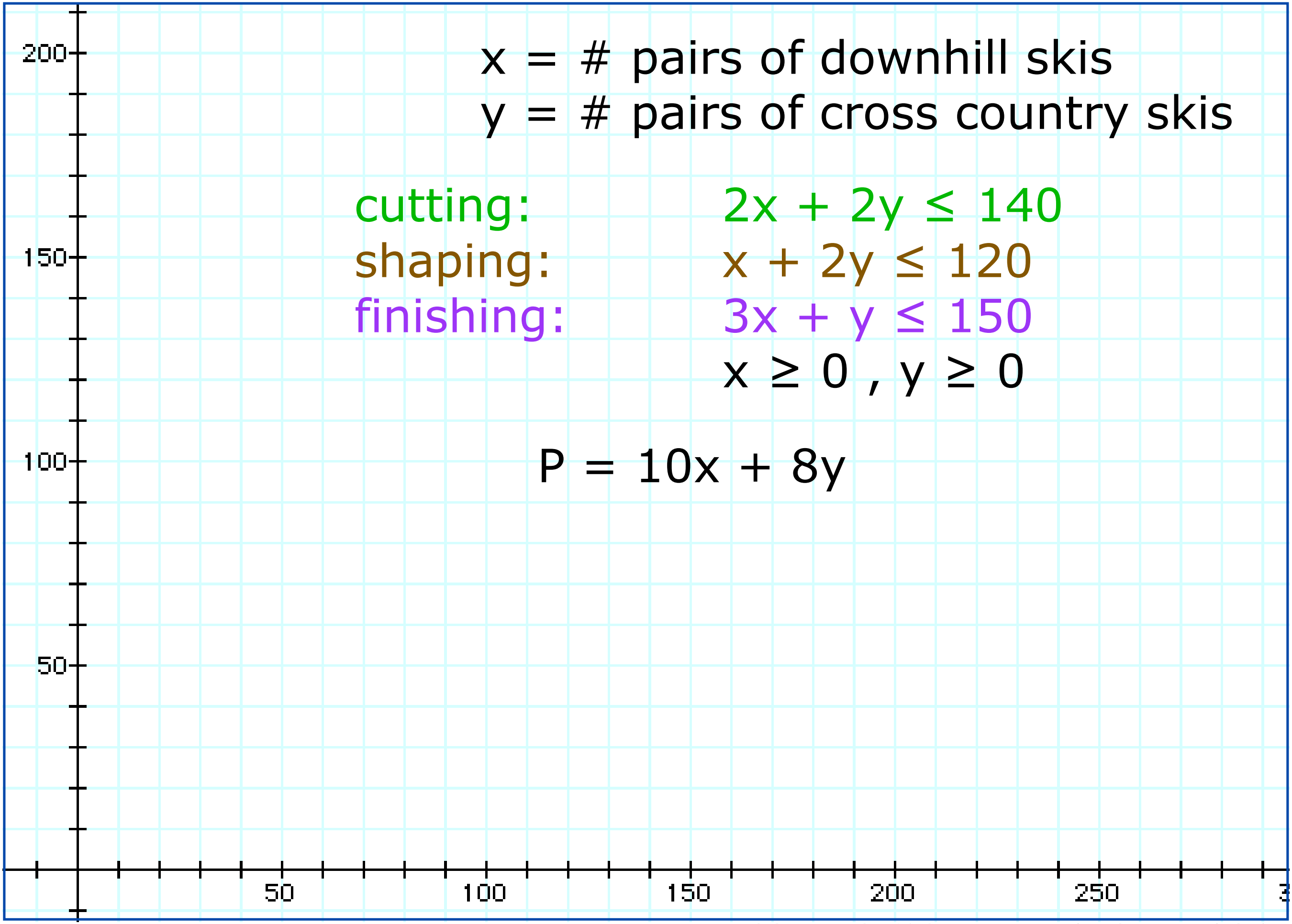
Example - ski manufacturing

Michigan Polar Products makes downhill and cross-country skis. A pair of downhill skis requires 2 man-hours for cutting, 1 man-hour for shaping and 3 man-hours for finishing while a pair of cross-country skis requires 2 man-hours for cutting, 2 man-hours for shaping and 1 man-hour for finishing. Each day the company has available 140 man-hours for cutting, 120 man-hours for shaping and 150 man-hours for finishing. How many pairs of each type of ski should the company manufacture each day in order to maximize profit if a pair of downhill skis yields a profit of $10 and a pair of cross-country skis yields a profit of $8?

x = # pairs of downhill skis
y = # pairs of cross country skis

cutting: $2x + 2y \le 140$
shaping: $x + 2y \le 120$
finishing: $3x + y \le 150$
$x \ge 0$, $y \ge 0$

$$P = 10x + 8y$$



x = # pairs of downhill skis
y = # pairs of cross country skis

cutting: $2x + 2y \le 140$
shaping: $x + 2y \le 120$
finishing: $3x + y \le 150$
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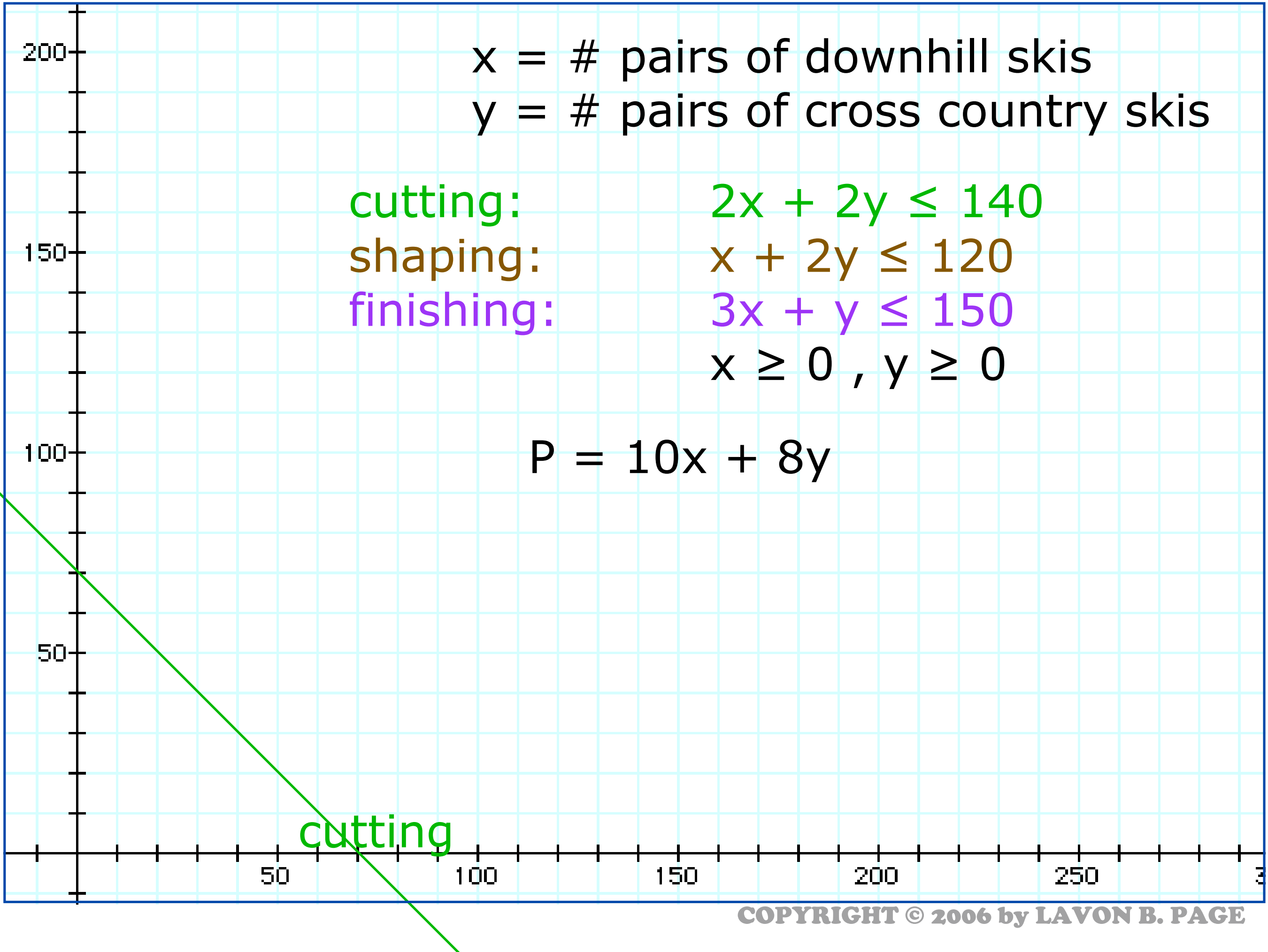


x = # pairs of downhill skis

y = # pairs of cross country skis

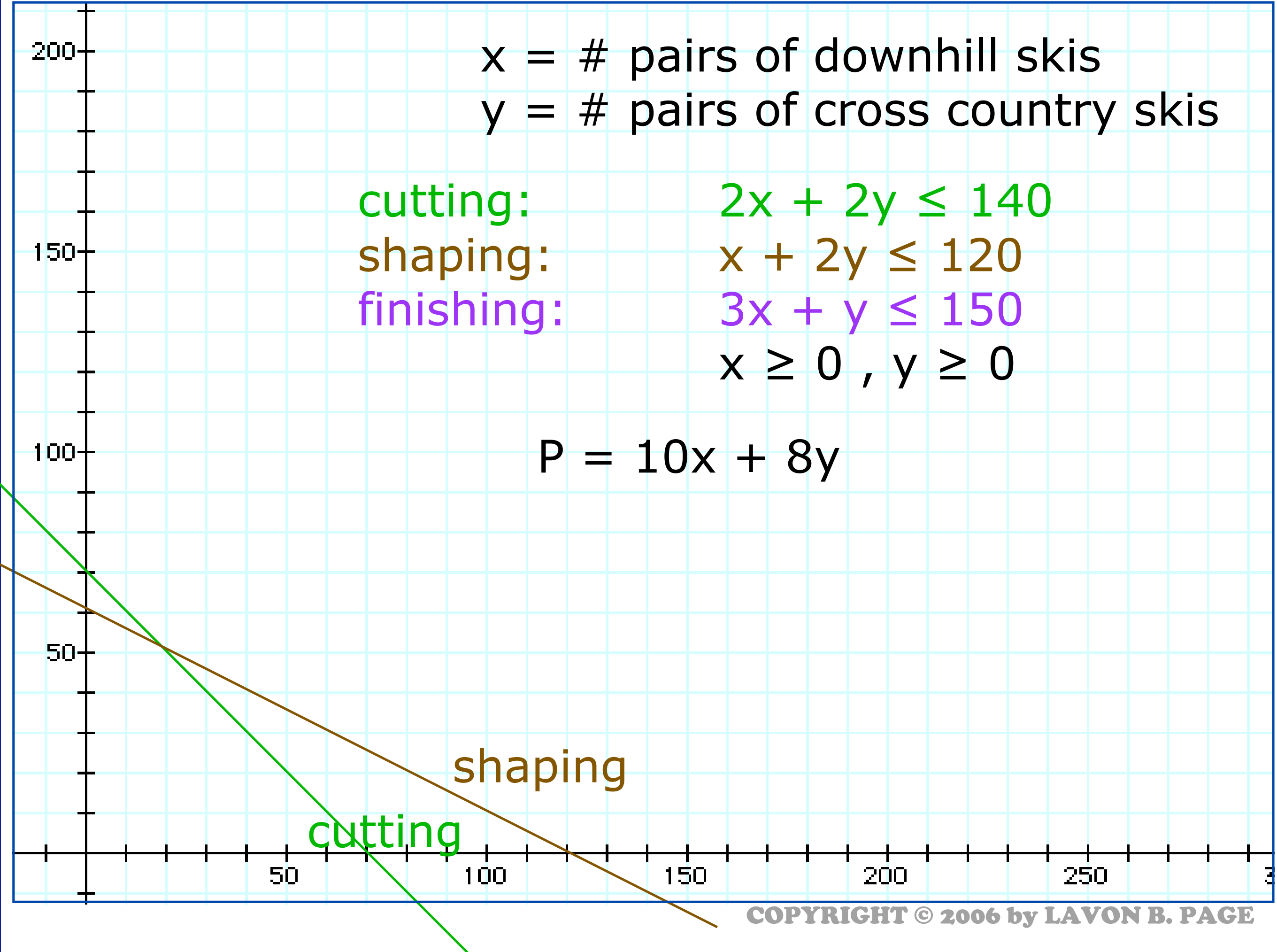
cutting: $2x + 2y \le 140$

shaping: $x + 2y \le 120$

finishing: $3x + y \le 150$

$x \ge 0\ ,\ y \ge 0$

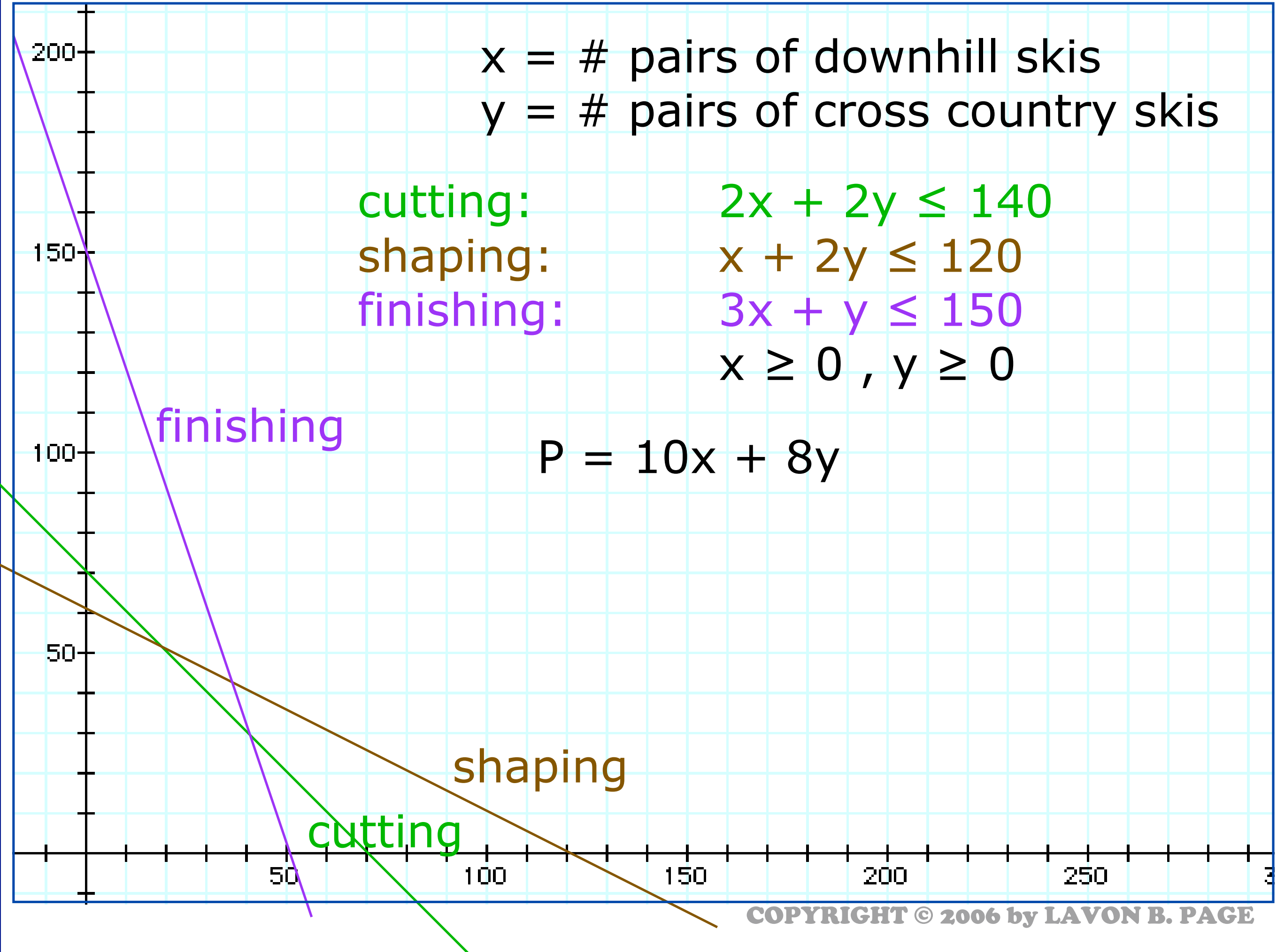
$P = 10x + 8y$

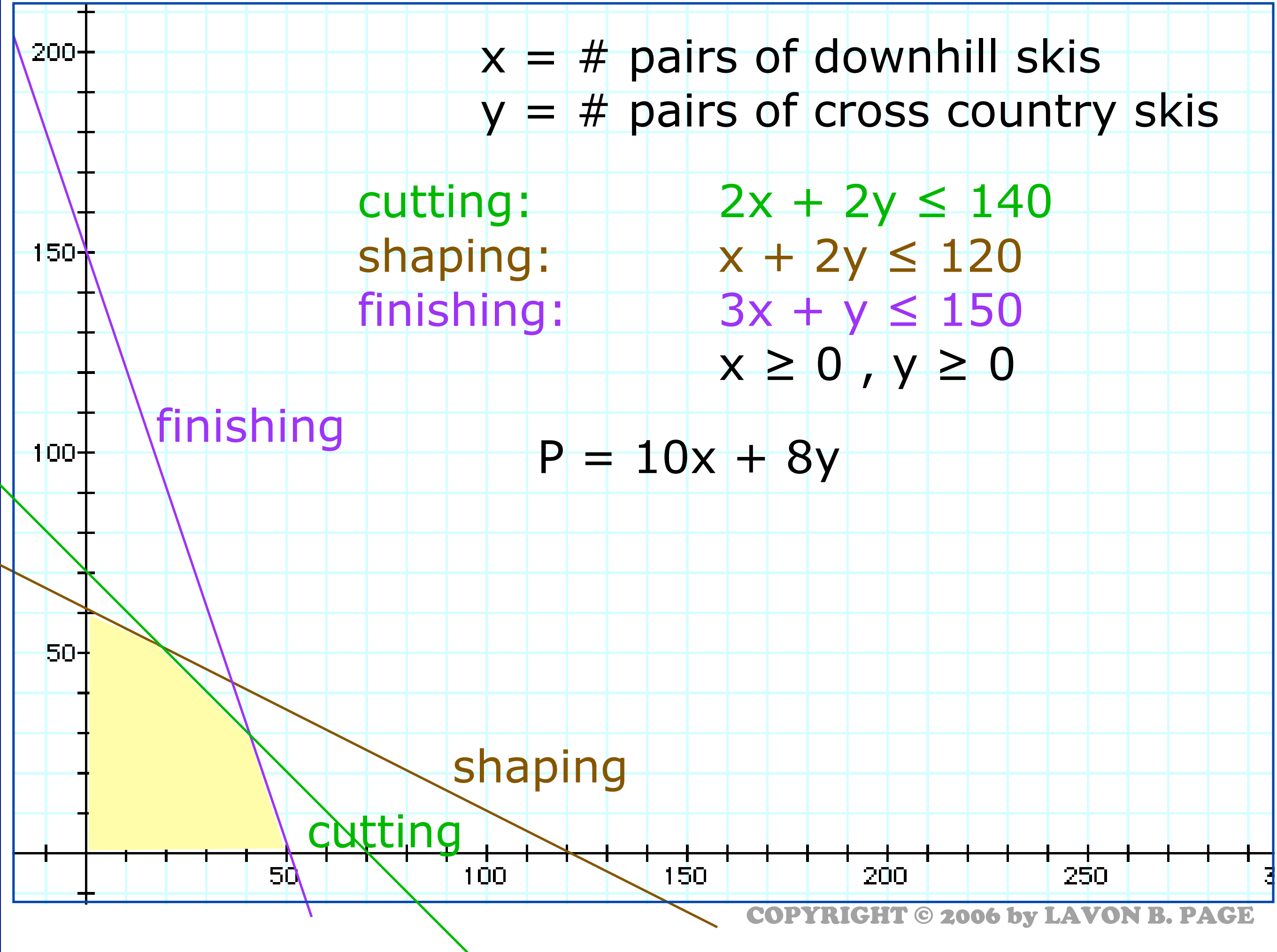


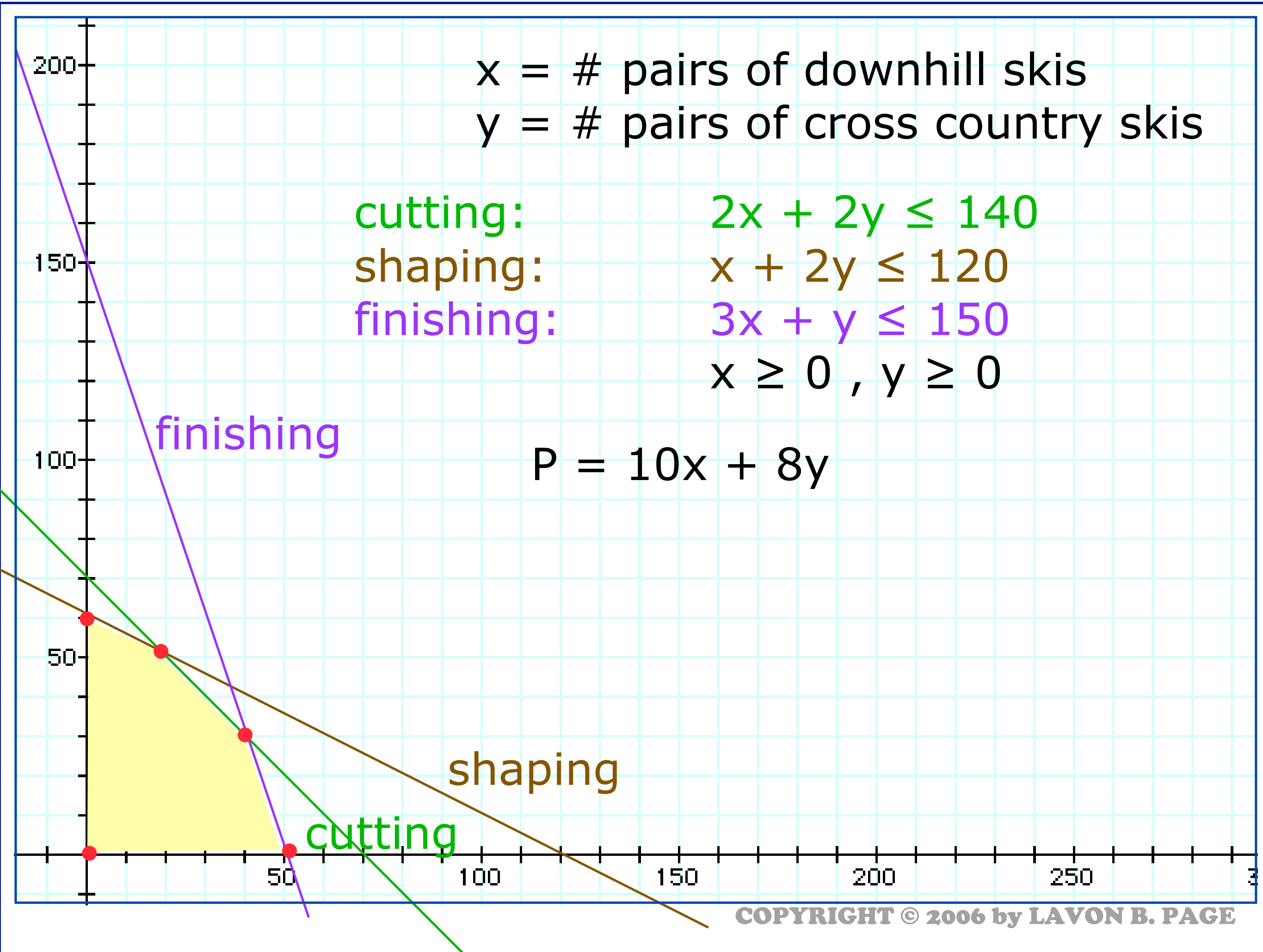
x = # pairs of downhill skis
y = # pairs of cross country skis

cutting: $2x + 2y \leq 140$
shaping: $x + 2y \leq 120$
finishing: $3x + y \leq 150$
$x \geq 0\ ,\ y \geq 0$

$P = 10x + 8y$




x = # pairs of downhill skis
y = # pairs of cross country skis
cutting: 2x + 2y ≤ 140
shaping: x + 2y ≤ 120
finishing: 3x + y ≤ 150
x ≥ 0 , y ≥ 0
P = 10x + 8y
finishing
shaping
cutting
200
150
100
50
50
100
150
200
250

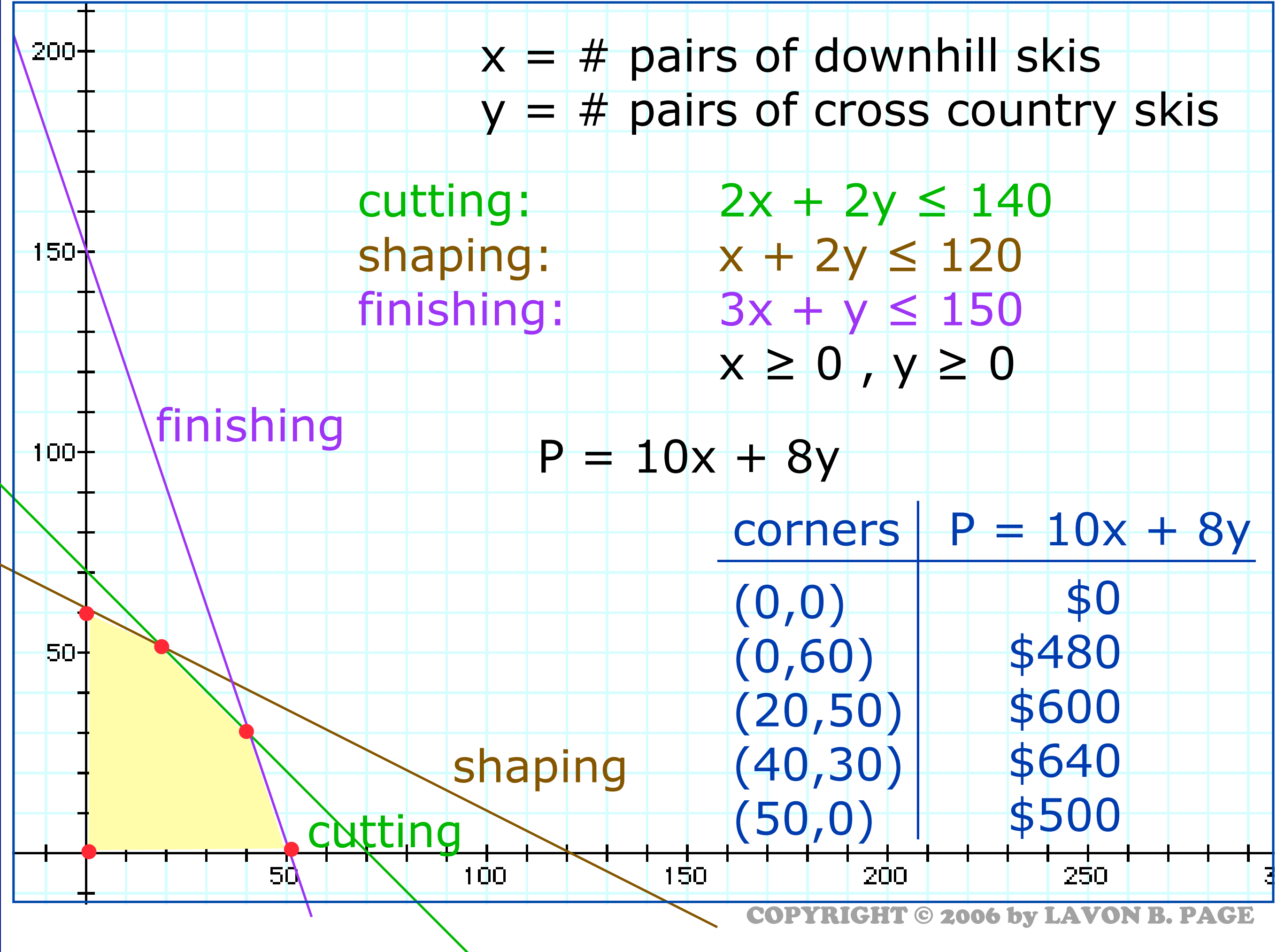

x = # pairs of downhill skis
y = # pairs of cross country skis
cutting: 2x + 2y ≤ 140
shaping: x + 2y ≤ 120
finishing: 3x + y ≤ 150
x ≥ 0 , y ≥ 0
P = 10x + 8y
finishing
shaping
cutting
200
150
100
50
50
100
150
200
250

x = # pairs of downhill skis
y = # pairs of cross country skis

cutting:	$2x + 2y \le 140$
shaping:	$x + 2y \le 120$
finishing:	$3x + y \le 150$
	$x \ge 0 \,,\; y \ge 0$

$$P = 10x + 8y$$

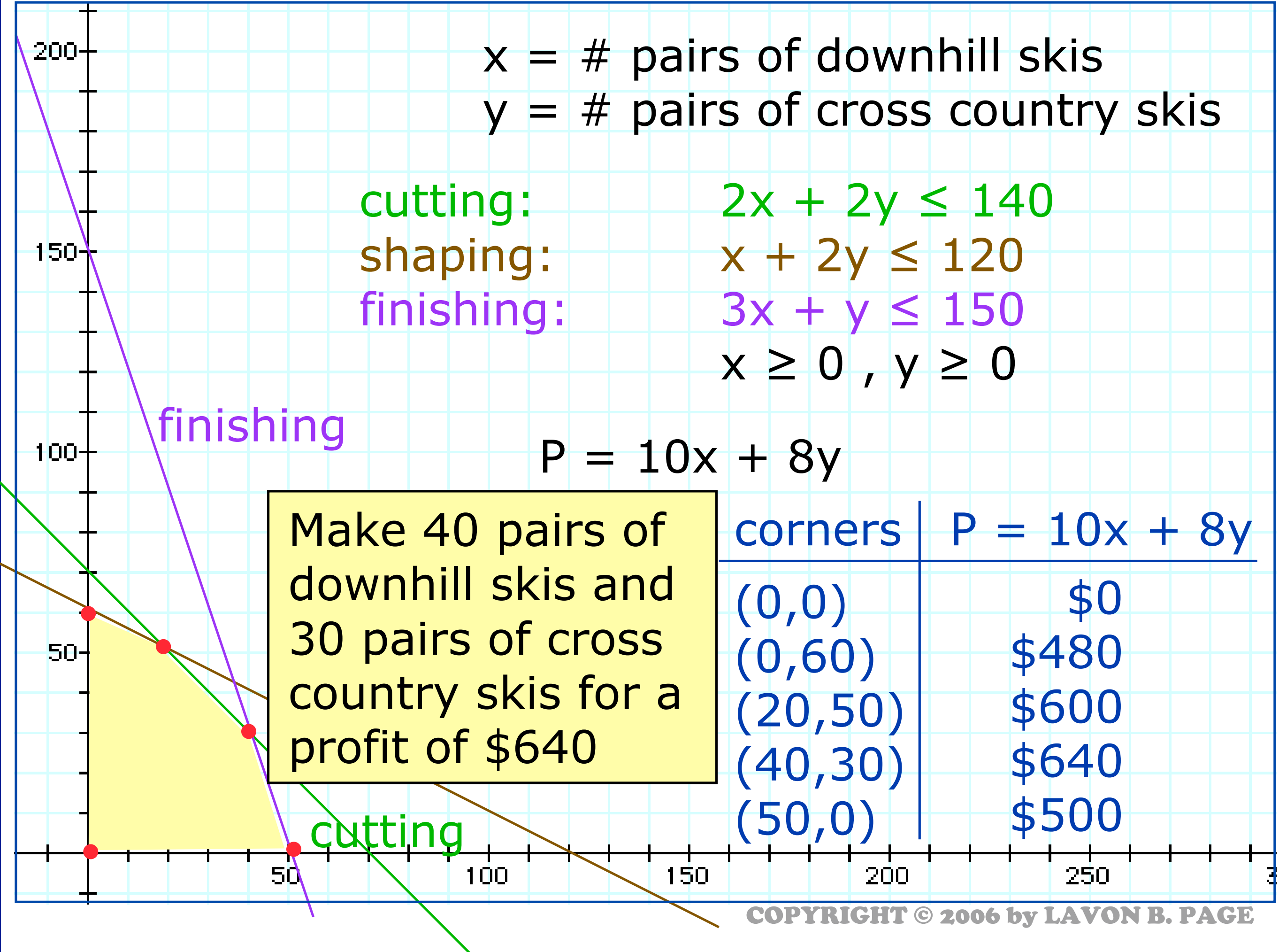
corners	$P = 10x + 8y$
(0,0)	\$0
(0,60)	\$480
(20,50)	\$600
(40,30)	\$640
(50,0)	\$500



x = # pairs of downhill skis
y = # pairs of cross country skis

cutting: $2x + 2y \le 140$
shaping: $x + 2y \le 120$
finishing: $3x + y \le 150$
$x \ge 0 , y \ge 0$

$P = 10x + 8y$



Make 40 pairs of downhill skis and 30 pairs of cross country skis for a profit of $640

corners	$P = 10x + 8y$
(0,0)	$0
(0,60)	$480
(20,50)	$600
(40,30)	$640
(50,0)	$500