Last Name **Name** **student ID (matricola)**

n = student ID No. (*N. matricola, per intero oppure cifre finali*)

Section 1: LINEAR PROGRAMMING

Given a set of foods, along with the nutrient information for each food and the cost per serving of each food, the objective of this diet problem is to select the number of servings of each food to purchase (and consume) so as to minimize the cost of the food while meeting the specified nutritional requirements.
There are 3 foods available: corn, milk, and bread.
There are restrictions on:

- the amount of Vitamin A: minimum 5000 units
- the number of calories: maximum 2250 units

Other constraints on the number of servings are included to improve the quality of the menu; therefore, the maximum number of servings is 10 per each specific food.
The table lists, for each food, the cost per serving, the amount of Vitamin A per each serving, and the number of calories per each serving.

Food	**Cost per serving**	**Vitamin A**	**Calories**
Corn	$0.18	107	72
2% Milk	$0.23	500	121
Wheat Bread	$0.05	0	65

Questions

a. Write the LP math model of this problem

b. Solve it and describe the approach to optimum step by step using the Matlab tools

c. Determine the optimal value of the **objective function**

d. Determine the optimal values of the **decision variables**

e. At the **optimum,** identify and list **basic** and **non-basic** variables, each with its final optimal value

f. Provide comments on special or unexpected features, if any, of the optimal solution

g. At the **optimum**, calculate the total amount of Vitamin A and the total number of calories

Section 2: EMPIRICAL MODELS

The attached file reports measured temperature data during the initial heating phase in a drying chamber for salami ripening:

Exam 2021-01-18_MMPIA.xlsx

By using Matlab® tools

a. determine the best **regression model**

b. report values for the **performance indexes** of the regression model

c. discuss **formulas and meaning** of the performance indexes

d. plot and discuss the **residuals**

e. using the best **regression model,** extrapolate the temperature to a new time 5 minutes after the last measured point

Section 3: FINITE DIFFERENCE METHODS for PDE

Solve the following parabolic PDE $$\frac{\partial u(x,t)}{\partial t} = \Delta \frac{\partial^2 u(x,t)}{\partial x^2} + k u(x,t)$$

with

$\Delta = (n-0.05)/(n+0.05)$

$k = 0$

$L = 10$

$t_{final} = 5$

IC: $t = 0$ $u(x,0) = \sin(x)$

BC: $$A \cdot u(x,t)\big|_{x=0} + B\frac{\partial u(x,t)}{\partial x}\bigg|_{x=0} = t$$

$$D \cdot u(x,t)\big|_{x=L} + E\frac{\partial u(x,t)}{\partial x}\bigg|_{x=L} = (t)^{\frac{1}{n}}$$

A=B=2

D=E= (n–0.05)/(n+0.05)

a. which type are the **Boundary Conditions**?

b. adopt the **explicit method** and, using **MUC**, explain the procedure briefly, attach the graph and comment the final solution

c. discuss the stability of the used method and specify the new value for the time-step if the explicit method turns out unstable

Section 4: MATHEMATICAL MODELING

4.1 Classification of a model

With ref. to the following model:

$$\frac{dx_1}{dt} = 1 + \frac{x_1}{1!} + \frac{{x_1}^2}{2!} + \frac{x_2}{3!}$$

$$\frac{dx_2}{dt} = 2 + \frac{x_1}{10} + \frac{x_1 x_2}{20} + \frac{x_2}{30}$$

$$with\ x_1 = x_1(t);\ x_2 = x_2(t)$$

$$IC: x_1(0) = 0.1;\ x_2(0) = 0.2$$

a) identify at least one general classification to which this model can be attributed

b) provide all attributes and categories you consider pertinent within the chosen classification

4.2 Dynamical models

a) Describe differences - conceptual and mathematical shape - between **continuous** and **discrete-time** dynamical models

A schematic and short exposition will be better evaluated !