
Last Name **Name** **student ID (matricola)**

n = _____ (student ID No. /// *N. matricola, per intero oppure cifre finali*)

Section 1: LINEAR PROGRAMMING

A confectionery company produces three types of sweets (say S1, S2 and S3) based on almonds, walnuts and candied fruit. The quantities of ingredients (expressed in grams per kg of cake) are shown in the following table.

Sweet	Almonds	Walnuts	Candy Fruits
	expressed in grams per kg of cake		
S1	200	150	50
S2	100	75	0
S3	0	125	40

For example, to produce 1 kg of sweet S1, it takes 200 g of almonds, 150 g of walnuts and 50 g of candy fruits.

Each working week, the availability of the ingredients is 14 kg of almonds, 16 kg of walnuts and 10 kg of candy fruits.

Each working week, 1 kg of sweet S1 must be produced at least.

At the end of each week, the quantities of almonds, walnuts and candy fruits that are not used for production of the three sweets are sold to the public at a price of 2, 3 and 1 euro/hg (hectogram), respectively.

The retail prices of the three sweets (say S1, S2 and S3) are respectively 15, 20 and 16 euro/kg.

Formulate the problem as an optimization problem, taking into account the constraints on weekly resources and with the aim of maximizing the weekly revenue from the sale of the three types of sweets and ingredients (almonds, walnuts and candy fruits) left unused in the production.

Questions

1.I. Formulate and write the LP math model of this problem

1.II. Solve it by using the most convenient tool in Matlab and describe **step by step** the obtainment of the **optimum**

1.III. Determine the optimal value of the **objective function**

1.IV. Determine the optimal values, if any, of the **decision variables**

- 1.V. At the **optimum**, provide comments on special or unexpected features, if any, e.g., regarding the role of the **decision variables**
- 1.VI. Coming back to the original problem, think and formulate an **additional constraint** that turns out **redundant**

Section 2: EMPIRICAL MODELS

The following (x,y) data were stored in the Matlab file

Sect.2_datikinsemp.mat

Questions

- 2.1.First, carefully look at data before using any Matlab® tool
- 2.2.plot such original data and attach the graph here
- 2.3.determine one **regression model** that you consider reasonably valid
- 2.4.is the regression model adopted by you a LINEAR or NON-LINEAR one?
- 2.5.calculate and discuss the **residuals**
- 2.6.plot the **residuals** as a **bar chart** of their distribution
- 2.7.using the predictions of the regression model adopted by you, plot the **Equivalent Graph (or Parity Line)**
- 2.8.using the regression model adopted by you, calculate a right-side **Extrapolated point** at new abscissa: $x=1800$ and then $x=1900$

2.9.using the regression model adopted by you, calculate a left-side **Extrapolated point** at new abscissa: $x=600$ and then $x=500$

Section 3: FINITE DIFFERENCE METHODS for PDE

Solve the 2nd order parabolic PDE with the following data:

$$\Delta = \left(\frac{n - 0.05}{n + 0.05} \right) \frac{1}{2}$$

$$k = 1$$

$$L = 5$$

$$t_{\text{final}} = 5$$

$$\text{IC: } t = 0 \quad u(x,0) = x^{3/2}$$

$$\text{BC: } A \cdot u(x,t) \Big|_{x=0} + B \frac{\partial u(x,t)}{\partial x} \Big|_{x=0} = \left(\frac{n-0.05}{n+0.05} \right)$$

$$D \cdot u(x,t) \Big|_{x=L} + E \frac{\partial u(x,t)}{\partial x} \Big|_{x=L} = e^{-\left(\frac{n-0.05}{n+0.05} \right) t}$$

$$A = D = \left(\frac{n-0.05}{n+0.05} \right)$$

where $n = \underline{\hspace{1cm}}$ (student ID No. /// N. matricola, per intero oppure cifre finali)

Questions

3.1. which type are the **Boundary Conditions**?

3.2. adopt the **explicit method** and, using **MUC**, explain the procedure **briefly**, attach the graph and comment the final solution

3.3. discuss the stability of the used method and specify the new value for the time-step if the explicit method turns out unstable

3.4. after that, change the BC by taking $A=B=D=E=\left(\frac{n-0.05}{n+0.05}\right)$, solve the PDE and compare the final profile to the previous case

Section 4: MATHEMATICAL MODELING

4.1 1st Classification of math models

- Describe the 1st classification by your own words
- Provide your favorite example of a math model in the 1st classification

Section 5: TIME SERIES

With ref. to the **time series** data in the Matlab file
Sect.5_dati5.mat

Questions

- plot the original **time series**

Then, calculate using Matlab

- standard deviation** of data
- skewness** of data
- kurtosis** of data

Then,

- identify the **outliers** in the original **time series**, exclude them by proposing a reasonable criterion to this end

Then, using the script *moving.m* or another Matlab tool

- propose a significant value of the **span M** for an effective **moving average**

- 5.g) obtain a new **filtered time series** from the original data by adopting the above **moving average** calculation
- 5.h) plot the new **filtered time series** against the original data and comment it