Last Name **Name** **student ID (matricola)**

n = _____ (student ID No. /// *N. matricola, per intero oppure cifre finali*)

Section 1: LINEAR PROGRAMMING

A cannery must produce fruit juice by mixing **fruit pulp** and **sweetener** to obtain a final product that must satisfy certain requirements regarding the content of vitamin C, mineral salts and sugar.
The fruit pulp and sweetener are purchased at a cost of respectively 2 € and 3 € per kilogram (kg).
Furthermore, the suppliers' labels state that 100 g (i.e., 1 hectogram hg) of fruit pulp contain 140 mg of vitamin C, 20 mg of mineral salts and 25 g of sugar, while 100 g of sweetener contain 10 mg of mineral salts, 50 g of sugar and do not contain vitamin C.
The requirements for the final product are the following: it must contain per kilogram (kg) of the fruit juice ready for sale

- at least 70 mg of vitamin C,
- at least 30 mg of mineral salts
- and at least 75 g of sugar.

The quantities of fruit pulp and sweetener must be determined to be used in the production of fruit juice in order to minimize the overall cost of purchasing those two basic components.

Questions

1.I. Formulate and write the LP math model of this problem

1.II. Solve it by using the most convenient tool in Matlab and describe **step by step** the obtainment of the **optimum**

1.III. Determine the optimal value of the **objective function**

1.IV. Determine the optimal values, if any, of the **decision variables**

1.V. At the **optimum**, provide comments on special or unexpected features, if any, e.g., regarding the role of the **decision variables**

1.VI. Coming back to the original problem, think and formulate an **additional constraint** that turns out **redundant**

Section 2: EMPIRICAL MODELS

The following (x,y) data were stored in the Matlab file

Sect.2_moulds_2014.xlsx

Questions

2.1.First, carefully look at data before using any Matlab® tool

2.2.plot such original data and attach the graph here

2.3.determine the **regression model** that you consider reasonably valid after taking in due consideration reasonable **Extrapolated Values** at the new abscissas: 20 and 25

2.4.is the regression model adopted by you a LINEAR or NON-LINEAR one?

2.5.calculate and discuss the **residuals**

2.6.plot the **residuals** as a **bar chart** of their distribution

2.7.using the predictions of the regression model adopted by you, plot the **Equivalent Graph (or Parity Line)**

2.8.using the regression model adopted by you, calculate an **Interpolated point** at a new abscissa: 5

Section 3: FINITE DIFFERENCE METHODS for PDE

Solve the 2nd order parabolic PDE with the following data:

$$\Delta = \left(\frac{n - 0.05}{n + 0.05}\right)\frac{1}{2}$$

k = 0

L = 10

t_{final} = 10

IC: t = 0 $u(x,0) = x^{3/2}$

BC: $A \cdot u(x,t)\big|_{x=0} + B\frac{\partial u(x,t)}{\partial x}\Big|_{x=0} = t\left(\frac{n-0.05}{n+0.05}\right)$

$D \cdot u(x,t)\big|_{x=L} + E\frac{\partial u(x,t)}{\partial x}\Big|_{x=L} = 2^{-\left(\frac{n-0.05}{n+0.05}\right)t}$

$A = D = \left(\frac{n-0.05}{n+0.05}\right)$

B = E = 0

where n = ___ (student ID No. /// *N. matricola, per intero oppure cifre finali*)

Questions

3.1. which type are the **Boundary Conditions**?

3.2. adopt the **explicit method** and, using **MUC**, explain the procedure briefly, attach the graph and comment the final solution

3.3. discuss the stability of the used method and specify the new value for the time-step if the explicit method turns out unstable

3.4. after that, adopt the **Crank-Nicholson method** and, using **MUC**, solve the PDE again and compare the final profile to the previous case

Section 4: MATHEMATICAL MODELING

4.1 3rd Classification of math models

a) Describe the above classification by your own words

b) Provide your favorite example of a math model in the above classification

Section 5: TIME SERIES

With ref. to the **time series** data in the Matlab file
Sect.5_Michelson's velocity.xlsx

Questions

5.a) plot the original **time series**

Then, calculate using Matlab

5.b) mean of data

5.c) standard deviation of data

Then, using the **Matlab Econometric Toolbox** or any other Matlab tool

5.d) Compute the sample autocorrelation function (ACF) of the time series

5.e) plot the correlogram of the time series and comment it

5.f) discuss the significance of the resulting Number of Lags