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Last Name	Name	student ID (matricola)
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n = \_\_\_\_\_ (student ID No. /// *N. matricola, per intero oppure cifre finali*)

## Section 1: LINEAR PROGRAMMING

A livestock industry dedicated to cattle breeding has set itself the objective of rationalizing the feeding of its herds in order to contain production costs.

Studies conducted by the veterinary sector have indicated that there are 3 basic nutritional elements (let's say A, B, C) and that the daily diet of each head of livestock must contain at least 27, 21, 30 units of A, B, C, respectively.

The farm is able to independently produce two types of fodder (let's indicate them as fodder 1 and fodder 2).

The analyses carried out on two types of fodder have provided the following results:

1 kg of fodder 1 contains 3, 1, 1 units of the nutritional elements A, B C, respectively;

1 kg of fodder 2 contains 1, 1, 2 units of the nutritional elements A, B C, respectively.

Bearing in mind that the production costs of the 2 types of fodder are 300 Lire/kg for fodder 1 and 150 Lire/kg for fodder 2, respectively, and wishing to respect the limits set on the minimum daily requirements for nutritional elements, the following is requested:

### Questions

1.I. Formulate and write the LP math model of this problem

1.II. Solve it by using the most convenient tool in Matlab and describe **step by step** the obtainment of the **optimum**

1.III. Determine the optimal values, if any, of the **decision variables**

1.IV. Determine the optimal value of the **objective function**

1.V. At the **optimum**, provide comments on special or unexpected features, if any, e.g., regarding the role of the **decision variables**

1.VI. Discuss how many variables and of which type (in LP terminology) would be necessary in the Simplex algorithm besides to the decision-making ones

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## Section 2: EMPIRICAL MODELS

The following (x,y) data were stored in the Matlab file

Sect.2 linreg.xmcd.txt

### Questions

2.1.First, carefully look at data, i.e., distance (mi) vs time (min), before using any Matlab® tool

2.2.plot such original data and attach the graph here

2.3.determine the **regression model** that you consider reasonably valid after taking in due consideration reasonable **Extrapolated Values** at the new abscissas: 70 and 85

2.4.is the regression model adopted by you a LINEAR or NON-LINEAR one?

2.5.calculate and discuss the **residuals**

2.6.plot the **residuals** as a **bar chart** of their distribution

2.7.using the predictions of the regression model adopted by you, plot the **Equivalent Graph (or Parity Line)**

2.8.using the same data, propose now an **interpolating model** of them

## Section 3: FINITE DIFFERENCE METHODS for PDE

Solve the 2<sup>nd</sup> order parabolic PDE with the following data:

$$\Delta = \left( \frac{n - 0.05}{n + 0.05} \right) \frac{1}{4}$$

$$k = 1$$

$$L = 1$$

$$t_{\text{final}} = 1$$

$$\text{IC: } t = 0 \quad u(x,0) = \sin(2x)$$

$$\text{BC: } A \cdot u(x, t)|_{x=0} + B \frac{\partial u(x,t)}{\partial x} \Big|_{x=0} = \sqrt{t \left( \frac{n-0.05}{n+0.05} \right)}$$

$$D \cdot u(x, t)|_{x=0} + E \frac{\partial u(x,t)}{\partial x} \Big|_{x=0} = \left( \frac{n-0.05}{n+0.05} \right) t$$

$$A = D = \left( \frac{n-0.05}{n+0.05} \right)$$

$$B = E = 0$$

where  $n = \underline{\hspace{1cm}}$  (student ID No. /// *N. matricola, per intero oppure cifre finali*)

## Questions

3.1. which type are the **Boundary Conditions**?

3.2. adopt the **Crank-Nicholson method** and, using **MUC**, explain the procedure briefly, attach the graph and comment the final solution

3.3. Change the BC to:

$$\text{BC: } A \cdot u(x, t)|_{x=0} + B \frac{\partial u(x,t)}{\partial x} \Big|_{x=0} = \sqrt{t \left( \frac{n-0.05}{n+0.05} \right)}$$

$$D \cdot u(x, t)|_{x=0} + E \frac{\partial u(x,t)}{\partial x} \Big|_{x=0} = \left( \frac{n-0.05}{n+0.05} \right) t$$

$$A = D = 0$$

$$B = E = \left( \frac{n-0.05}{n+0.05} \right)$$

3.4. While keeping the **Crank-Nicholson method** and using **MUC**, attach the new graph and comment the final solution

3.5. compare the final profiles of the above two cases

## Section 4: MATHEMATICAL MODELING

### 4.1 Math models

A mould population evolution is described in 2 different ways by the following 2 eqs. A) and B):

A)

$$N_0(t)\delta(d - d_0) - \mathfrak{R}(d) \frac{\partial[N(d, t)]}{\partial d} - N(d, t) \frac{\partial[\mathfrak{R}(d)]}{\partial d} = \frac{\partial N(d, t)}{\partial t}$$

IC: at  $t=0$   $N(d,0) = 0 \rightarrow$  No moulds population is initially present with a diameter  $d$

BC: at  $d=0, \forall t N(0,t) = 0 \rightarrow$  No colony may exist with a null diameter

B)

$$\dot{N} = cN(t) - hN^2(t) \quad \text{with } c > 0; h > 0$$

IC:  $N(t_0) = N_0$

- a) Describe the conceptual difference by your own words

## Section 5: TIME SERIES

With ref. to the **time series** data in the Matlab file

Sect.5 Corr.xmcd.xlsx

### Questions

- 5.a) plot the original **time series**

Then, calculate using Matlab

- 5.b) mean of data

- 5.c) standard deviation of data

Then, using the **Matlab Econometric Toolbox** or any other Matlab tool

- 5.d) Compute the sample autocorrelation function (ACF) of the time series

- 5.e) plot the correlogram of the time series and comment it

- 5.f) propose a significant **autocorrelation lag** and explain it