Last Name **Name** **student ID (matricola)**

n = _____ (student id no. /// *n. matricola, per intero oppure cifre finali*)

Section 1: LINEAR PROGRAMMING

A tomato processing and canning factory uses steam generated by its own boiler-house.
Consider the problem of minimizing the fuel costs in the boiler-house, which consists of two turbine generators (G_1, G_2).
The generators have different operating characteristics but can be simultaneously operated both with two fuels:

1. Fuel oil
2. Medium BTU gas (MBG)

The fuel oil is commercially available with no limitations but has a cost of 1000 US$/ton.
MBG is produced as a waste off-gas from another part of the factory and is made available for the boiler-house with a maximum flow rate of 5 ton/h; if it is not used on site, it must be flared. For such a reason it has a trivial cost of 0.1 US$/ton.

Past data collected on the fuel requirements for the generators yield the following empirical relations:
$P_1 = 4.5x_1 + 4x_2$ [MW]
$P_2 = 3.2x_3 + 2x_4$ [MW]
where P_1 is the power output from G_1, P_2 is the power output from G_2; x_1 is the fuel oil flow rate [ton/h] and x_2 [ton/h] is the MBG flow rate to G_1; x_3 is the fuel oil flow rate [ton/h] and x_4 [ton/h] is the MBG flow rate to G_2.
Also, the power generation from each unit is constrained:
$18 \leq P_1 \leq 30$ [MW]
$14 \leq P_2 \leq 25$ [MW]
The operating objective of the boiler-house is to provide at least 50 MW of power while minimizing costs.
The factory director wants to find the optimum flow rates of fuel oil and MBG.

Questions

1.I. Formulate and write the LP math model of this problem

1.II. Solve it by using the most convenient tool in Matlab and describe **step by step** the obtainment of the **optimum**

1.III. Determine the optimal value of the **objective function**

1.IV. Determine the optimal values, if any, of the **decision variables**

1.V. At the **optimum**, provide comments on special or unexpected features, if any, e.g., regarding the role of the **decision variables**

Section 2: EMPIRICAL MODELS

The vapor pressure (different units) data of water in the temperature range 0–100 °C are:

***T*, °C**	***T*, °F**	***P*, kPa**	***P*, torr**	***P*, atm**
0	32	0.6113	4.5851	0.0060
5	41	0.8726	6.5450	0.0086
10	50	1.2281	9.2115	0.0121
15	59	1.7056	12.7931	0.0168
20	68	2.3388	17.5424	0.0231
25	77	3.1690	23.7695	0.0313
30	86	4.2455	31.8439	0.0419
35	95	5.6267	42.2037	0.0555
40	104	7.3814	55.3651	0.0728
45	113	9.5898	71.9294	0.0946
50	122	12.3440	92.5876	0.1218
55	131	15.7520	118.1497	0.1555
60	140	19.9320	149.5023	0.1967
65	149	25.0220	187.6804	0.2469
70	158	31.1760	233.8392	0.3077
75	167	38.5630	289.2463	0.3806
80	176	47.3730	355.3267	0.4675
85	185	57.8150	433.6482	0.5706
90	194	70.1170	525.9208	0.6920
95	203	84.5290	634.0196	0.8342
100	212	101.3200	759.9625	1.0000

Questions

First, carefully look at data.
Then, use the Matlab® tools to manage them.

2.1.determine one **interpolation model** that you consider reasonably valid

2.2.determine one **regression model** that you consider reasonably valid

2.3.compare the **regression model** against the **interpolation model** and provide adequate comments

2.4.is the **regression model** adopted by you a LINEAR or NON-LINEAR one?

2.5.calculate and discuss the **residuals**

2.6.plot the **residuals** as a **bar chart** of their distribution

2.7.using the predictions of the regression model adopted by you, plot the **Equivalent Graph (or Parity Line)**

2.8.using the regression model adopted by you, calculate the **Extrapolated point** at a new abscissa of 77.5 °C

Section 3: FINITE DIFFERENCE METHODS for PDE

You are assigned the following parabolic PDE $\frac{\partial u(x,t)}{\partial t} = \Delta \frac{\partial^2 u(x,t)}{\partial x^2} + k u(x,t)$

with

$\Delta = n/2$

$k = n/4$

$L = 12$

$t_{final} = 5$

IC: $t = 0$ $u(x,0) = \sin(x)$

BC: $A \cdot u(x,t)|_{x=0} + B \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0} = (n+1)/n$

$D \cdot u(x,t)|_{x=L} + E \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L} = \sqrt{\frac{t}{n}}$

A=B=D=E=n

where n = _____ (student ID No. /// *N. matricola, per intero oppure cifre finali*)

Questions

3.1.which type are the **Boundary Conditions**?

3.2.adopt the **explicit method** and, using **MUC**, explain the procedure briefly,

3.3.attach the graph at t = 0

3.4.attach the final graph and comment the final solution

3.5.What is the role of Delta_x? How much is Delta_x ?

3.6.What is the role of Delta_t? How much is Delta_t ?

Section 4: MATHEMATICAL MODELING

4.1 Classification of a model

Look at the below model:

$$\dot{y}_1 = (1 - \frac{y_2}{\mu_2})y_1$$
$$\dot{y}_2 = -(1 - \frac{y_1}{\mu_1})y_2$$

a) Which math model is this?

b) Provide all possible classifications for it

c) How many and what are the parameters?

Section 5: TIME SERIES

With ref. to the **time series** data in the data file

sect.5.2024-09-04.txt

Questions

Calculate

a) mean

b) standard deviation (of the sample)

c) skewness

d) kurtosis

e) identify the **outliers** in the original **time series** and explain the criterion/tool you adopt to reasonably recognize/exclude them

f) obtain a new **time series** by removing/replacing the **outliers** from the original one

g) propose a significant value of the **span** M for a **moving average** calculation

h) obtain a new **filtered time series** from the previous data by adopting the above **moving average** calculation

i) plot the new **filtered time series** against the previous data and comment it

NB:
You may use whatever Matlab tool and the script *moving.m*