Last Name	Name	student ID (matricola)
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n = \_\_\_\_\_ (student ID No. /// N. matricola, per intero oppure cifre finali)

### **Section 1: LINEAR PROGRAMMING**

A farm is struggling with the completion between food (corn and wheat) and non-food (i.e., cotton) crop raising, while having the following resources available:

- $\Rightarrow$  100 acres of land for all crops
- $\Rightarrow$  500 hours of labor for manpower

Therefore, the farm is looking for a simple model to maximize the profit.

Based on past experience, the farm has the following data:

DATA	UNIT	corn	wheat	cotton
Profit per acre (cultivated)	US\$/acre	109	90	115
Workload per acre (cultivated)	h/acre	6	4	8

#### NB:

the farm knows in advance that must subtract 450 US\$ from the overall profit due to unavoidable land fees.

### Questions

- 1.I. Formulate and write the LP math model of this problem
- 1.II. Which **reference type** of LP problem does this one appear to be?
- 1.III. Solve it by using the most convenient tool in Matlab and describe step by step the obtainment of the **optimum**
- 1.IV. Determine the optimal value of the **objective function**
- 1.V. Determine the optimal values, if any, of the **decision variables**
- 1.VI. At the **optimum**, provide comments on special or unexpected features, if any, e.g., regarding the role of the **decision variables**

1.VII. As a possible complication of the given problem, provide – at least qualitatively – a further constraint that might be added to those on land and labor

### **Section 2: EMPIRICAL MODELS**

The following attached file contains data representing the time evolution of a Reactor Temperature (with fluidized coarse sand):

Sect.2\_2025-02-18 Reactor Test w coarse sand.TXT

NB:

Please note that the Reactor Temperature is expected to asymptotize Tr=200 °C.

# **Questions**

First, carefully look at data and then use Matlab® tools:

- 2.1.try a **regression model** that you consider reasonably valid, by using the Matlab resources to provide a solution
- 2.2.is the **regression model** chosen by you a LINEAR or NON-LINEAR one?
- 2.3.calculate and plot the **residuals** of the **regression model**
- 2.4. discuss the quality of the residuals by at least one of the methods presented in our course
- 2.5.using the predictions of the regression model adopted by you, plot the **Equivalent Graph** (or **Parity Line**)
- 2.6.using the regression model adopted by you, calculate the **Accuracy Factor**

# Section 3: FINITE DIFFERENCE METHODS for PDE

Solve the following parabolic PDE

$$\frac{\partial u(x,t)}{\partial t} = \Delta \frac{\partial^2 u(x,t)}{\partial x^2} + ku(x,t)$$

with

$$\Delta = \left(\frac{n - 0.05}{n + 0.05} \cdot \frac{10}{3}\right)$$

$$k = 0$$

$$L = 10$$

TIMESTEP = 0.5

IC: 
$$t = 0$$
  $u(x,0) = 5-x/2$ 

BC: 
$$A \cdot u(x,t)|_{x=0} + B \frac{\partial u(x,t)}{\partial x}|_{x=0} = 10 \cdot \left(\frac{n-0.05}{n+0.05}\right)$$

$$D \cdot u(x,t)|_{x=L} + E \frac{\partial u(x,t)}{\partial x}\bigg|_{x=L} = 0$$

$$A=D=1$$

$$B=E=0$$

where n = \_\_\_\_\_ (student ID No. /// N. matricola, per intero oppure cifre finali)

### Questions

- 3.1. Adopt the **explicit method** and, using MUC, check the stability of the method
- 3.2. Using **MUC**, attach the necessary graphical views to show (and discuss) the issue of "**physical incongruence**" exhibited by the explicit method
- 3.3. Switching to Crank&Nicholson, attach the necessary graphical views from MUC to demonstrate that the **Crank&Nicholson method** is not affected by "**physical incongruence**"

### **Section 4: MATHEMATICAL MODELING**

#### 4.1 Classification of a model

Look at the below model:

$$\dot{y}_1 = (1 - \frac{y_2}{\mu_2})y_1$$
  
 $\dot{y}_2 = -(1 - \frac{y_1}{\mu_1})y_2$ 

- a) Which math model is this?
- b) Provide all possible classifications for it
- c) How many and what are the parameters?

### **Section 5: TIME SERIES**

With ref. to the **time series** data in the file

Sect.5\_2025-02-18.txt

# **Questions**

First, carefully look at data and then use Matlab® tools. You may use whatever Matlab tool and the script *moving.m* 

#### Calculate

- 5.a. mean
- 5.b. standard deviation (of the sample)
- 5.c. skewness
- 5.d. kurtosis
- 5.e. propose a significant value of the span M for a moving average calculation
- 5.f. build a **filtered time series** which you may consider practically free of high-frequency oscillations
- 5.g. which value have you used for the **span** M in the previous step?

5.h. propose a reasonably different value for the **span** M, obtain a new **filtered time series** against the previous one and **comment** it