
Last Name	Name	student ID (matricola)
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n = _____ (student ID No. /// N. matricola, per intero oppure cifre finali)

Section 1: LINEAR PROGRAMMING

A company distributes three types of cookies (**A**, **B**, and **C**) from a central warehouse to two retail stores (**Store 1** and **Store 2**).

There are **Supply Constraints** due to limited **Warehouse Capacity**. The warehouse has a total supply of:

- **A**: 40 units
- **B**: 50 units
- **C**: 60 units

There are **Demand Constraints** due to **Store Requirements**.

Store 1 requires:

- At least **20** units of **A**
- At least **30** units of **B**
- At least **25** units of **C**

Store 2 requires:

- At least **15** units of **A**
- At least **20** units of **B**
- At least **30** units of **C**

The company wants to **minimize transportation costs** while ensuring demand is met at each store. The transportation costs per unit are in the following table.

Cookies	Store 1 (US \$/unit)	Store 2 (US \$/unit)
A	3	4
B	2	3
C	5	4

Questions

1.I. Formulate and write the LP math model of this problem.

1.II. What type of LP problem is this?

1.III. Solve it by using the most convenient tool in Matlab and describe **step by step** the obtainment of the **optimum**

1.IV. Determine the optimal value of the **objective function**

1.V. Determine the optimal values, if any, of the **decision variables**

1.VI. At the **optimum**, provide comments on special or unexpected features, if any, e.g., regarding the role of the **decision variables**

Section 2: EMPIRICAL MODELS

A farming cooperative gathered results on **wheat yield** (in tons per hectare) and wants to predict it based on data of **yearly rainfall** (in mm) and **yearly average temperature** (in °C).

All the data for the past **10 years** on **rainfall**, **temperature**, **wheat yield** are in the following Table:

Year	Rainfall (mm)	Temperature (°C)	Wheat Yield (tons/ha)
2014	800	18	3.2
2015	750	19	3.0
2016	820	17	3.4
2017	780	20	NA
2018	850	16	3.6
2019	790	21	3.0
2020	810	18	3.3
2021	830	17	3.5
2022	770	19	3.2
2023	860	15	3.8

NA = Not Available

Questions

2.1.plot the observations (**wheat yield**) as a function of **time** (year)

2.2.interpolate the observations (**wheat yield**) for the missing **time** (year)

Then, save the value interpolated above (**wheat yield**), use it and switch to regression techniques.

2.3.determine a **regression model** that at you consider reasonably valid as a function of **Rainfall** (mm) only

2.4. Report and comment the regression Performance Metrics (R^2 score, etc.)

2.5. calculate and discuss the **residuals**

2.6. determine a **regression model** that at you consider reasonably valid as a function of **Temperature ($^{\circ}\text{C}$)** only

2.7. Report and comment the regression Performance Metrics (R^2 score, etc.)

2.8. calculate and discuss the **residuals**

2.9. using the predictions of this last regression model, plot the **Equivalent Graph (or Parity Line)**

Section 3: FINITE DIFFERENCE METHODS for PDE

You are assigned the following parabolic PDE

$$\frac{\partial u(x,t)}{\partial t} = \Delta \frac{\partial^2 u(x,t)}{\partial x^2} + ku(x,t)$$

with

$$\Delta = (4n+4)/n$$

$$k = (n+1)/(4n)$$

$$L = 2\pi$$

$$t_{\text{final}} = 5$$

IC: $t = 0 \quad u(x,0) = \sin(x)$

BC: $A \cdot u(x,t)|_{x=0} + B \frac{\partial u(x,t)}{\partial x} \Big|_{x=0} = \frac{n+1}{4n}$

$$D \cdot u(x,t)|_{x=L} + E \frac{\partial u(x,t)}{\partial x} \Big|_{x=L} = \sqrt{\frac{n+1}{4n}} t$$

$$A = D = 0$$

$$B = E = 1$$

where $n = \underline{\hspace{1cm}}$ (student ID No. /// *N. matricola, per intero oppure cifre finali*)

Questions

3.1. which type are the **Boundary Conditions**?

3.2. adopt the **explicit method** and, using **MUC**, explain the procedure briefly

3.3. attach the graph at $t = 0$

3.4. What is the role of Δx ? How much is Δx ?

3.5. What is the role of Δt ? How much is Δt ?

3.6. attach the final graph at t_{final} and comment the final solution

Section 4: MATHEMATICAL MODELING

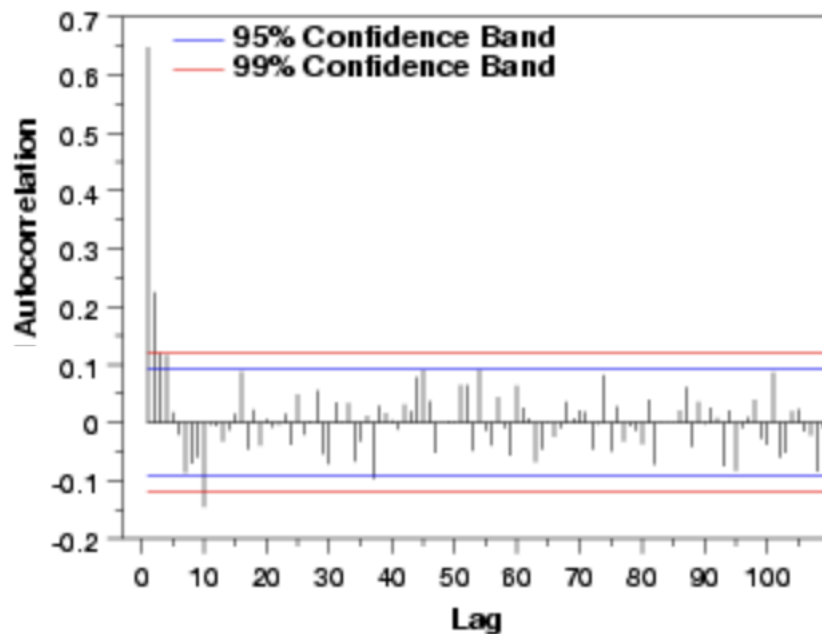
4.1 First principles models

- a) What is the difference between **Basic models** and **Models involving transport phenomena**?
- b) Give at least 3 examples of a “**generation term**” in First principles models

Section 5: TIME SERIES

5.1 Time series analysis

Consider the following plot:



- 5.a. Provide a general description/comment of it.
- 5.b. Is the **time series** showing any autocorrelation?
- 5.c. If yes, what is the resulting **Number of Lags**?

5.2 Autocorrelation

Consider the data in the file

Sect.5.2_Y.txt

being a **time series**.

Then, using the Matlab Econometric Toolbox:

- 5.d. compute the sample **autocorrelation function** (ACF) of the time series.
- 5.e. plot the **correlogram** of the time series and **comment** it
- 5.f. make one or more reasonable choices of the **Number of Lags** and discuss the resulting significance