Last Name **Name** **student ID (matricola)**

n = _____ (student ID No. /// *N. matricola, per intero oppure cifre finali*)

Section 1: LINEAR PROGRAMMING

A company starts the experimental production of two kinds of specialty cookies: **Cookie A** and **Cookie B**.
Among the other resources, their production involves two types of sugars: **1) White Sugar** and **2) Cane sugar**, with the following recipe requirements:

- **1) White Sugar**: one Cookie A requires 2 g, and one Cookie B requires 1 g.
- **2) Cane sugar**: one Cookie A requires 1 g, and one Cookie B requires 2 g.

The company has available for such an experimental production a total of 100 g of **1) White Sugar** and 80 g of **2) Cane sugar**.
The experimental production becomes feasible if and only if at least 10 **Cookie A** and at least 9 **Cookie B** are produced.
The company wants to maximize its profit, which is expected to derive from a selling price of 40 $/cookie (from Cookie A) and 30 $/cookie (from Cookie B), while a fixed amount of 300 $ is to be subtracted as General Expenses.

Questions

1.I. Formulate and write the LP math model of this problem.

1.II. What type of LP problem is this❓

1.III. Solve it by using the most convenient tool in Matlab and describe `step by step` the obtainment of the **optimum**

1.IV. Determine the optimal value of the **objective function**

1.V. Determine the optimal values, if any, of the **decision variables**

1.VI. What should the selling prices be to have infinite solutions at the **optimum**❓

Section 2: EMPIRICAL MODELS

The following attached file contains data representing the time evolution of a microorganism growth:
Sect.2_microorganism.TXT

NB:
Please note that the microorganism count is expected to asymptotize the value 8.6!

Questions

First, carefully look at data and then use Matlab® tools:

2.1. try a **regression model** that you consider reasonably valid, by using the Matlab resources to provide a solution

2.2. is the **regression model** chosen by you a LINEAR or NON-LINEAR one?

2.3. Report and comment the regression Performance Metrics (R^2 score, etc.)

2.4. calculate and plot the **residuals** of the **regression model**

2.5. discuss the **quality of the residuals** by using and showing the **Normal Probability Plot**

2.6. using the predictions of the regression model adopted by you, plot the **Equivalent Graph (**or **Parity Line)**

2.7. using the regression model adopted by you, calculate the **Accuracy Factor**

Section 3: FINITE DIFFERENCE METHODS for PDE

You are assigned the following parabolic PDE $\frac{\partial u(x,t)}{\partial t} = \Delta \frac{\partial^2 u(x,t)}{\partial x^2} + ku(x,t)$

with

$\Delta = (4n+4)/n$

$k = (n+1)/(4n)$

L = 2π

t_{final} = 5

IC: t = 0 u(x,0) = sin(x)

BC: $A \cdot u(x,t)|_{x=0} + B \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=0} = \frac{n+1}{4n}$

$D \cdot u(x,t)|_{x=L} + E \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=L} = \frac{n+1}{4n}\sqrt{t}$

A = D = 0

B = E =1

where n = _____ (student ID No. /// *N. matricola, per intero oppure cifre finali*)

Questions

3.1.which type are the **Boundary Conditions**?

3.2.Chose a value of **Delta_t** such as the **explicit method**, using **MUC**, remains always stable for 3 cases:

#POINTS = 5

#POINTS = 10

#POINTS = 20

3.3.attach the final graph at t_{final} as obtained with #POINTS = 5

3.4.attach the final graph at t_{final} as obtained with #POINTS = 10

3.5. attach the final graph at t_{final} as obtained with #POINTS = 20

3.6.comment and compare the 3 final solutions

Section 4: MATHEMATICAL MODELING

4.1 Model properties

a) What order is the following model❓

$$a_1\left(\frac{dy}{dt}\right)^2 + a_2 y(t) = a_3 t^3$$

b) Is the **phase portrait** of such a dynamical system located on mono- (straight line), bi- (plane) or tri-dimensional space❓

4.2 Lumped vs Distributed parameters models

c) Discuss the difference

Section 5: TIME SERIES

With ref. to the **time series** data in the file:

Sect.5_detrend.txt

and the use of Matlab®,
first perform and comment the **exploratory data analysis**:

5.a. plot the original **time series**

5.b. Calculate standard deviation

5.c. Calculate skewness

5.d. Calculate kurtosis

Then, using the Matlab® command:

```
y = detrend(x,n)
% removes a polynomial trend with degree n from the data in
vector x and returns the residual in vector y.
n = 1 by default.
Setting n = 0 is equivalent to using the 'constant' option and
setting n = 1 is equivalent to using the 'linear' option.
```

or alternatively the *Matlab Econometric Toolbox*:

5.e. Obtain the best "**detrended**" time series

5.f. Plot the new "**detrended**" time series against the original data and comment it

Finally, start a search for any reasonable **outlier**:

5.g. Identify the **outliers** and discuss the criterion you've used for this task

5.h. Replot the new "**detrended**" time series free of **outliers**