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Last Name	Name	student ID (matricola)
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n = student ID No. (*N. matricola, per intero oppure cifre finali*)

## Section 1: LINEAR PROGRAMMING

A plastic recycling plant produces two products A and B, whose daily production cost per quintal is equal to 200 and 300 €/q, respectively.

Each day a maximum of 8 quintals must be produced of A and B together, from which at least 2 q of product A and a maximum of 5 q of product B.

Furthermore, the ratio between the daily productions of A and B must be below a maximum of 3.

What are the daily productions expressed in quintals of A and B, what minimize the daily cost of total production?

## Questions

1.I. Formulate and write the LP math model of this problem

1.II. What type of LP problem is this ?

1.III. Solve it by using the most convenient tool in Matlab and describe **step by step** the obtainment of the **optimum**

1.IV. Determine the optimal value of the **objective function**

1.V. Determine the optimal values, if any, of the **decision variables**

1.VI. At the **optimum**, provide comments on special or unexpected features, if any

## Section 2: EMPIRICAL MODELS

In an experiment, the data was obtained in the following file.

sect.2\_concentration data.txt

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## Questions

First, carefully look at data and then use Matlab® tools:

- 2.1.determine one **regression model** that you consider reasonably valid
- 2.2.is the regression model adopted by you a LINEAR or NON-LINEAR one?
- 2.3.calculate and plot the **residuals**
- 2.4.discuss the **residuals**
- 2.5.using the predictions of the regression model adopted by you, plot the **Equivalent Graph (or Parity Line)**
- 2.6.using the regression model adopted by you, calculate the **Interpolated point** at an abscissa of 7.5
- 2.7.using the regression model adopted by you, calculate the **Extrapolated point** at a new abscissa of 12
- 2.8.propose and determine another regression model that you consider “**overfitting**” the data

## Section 3: FINITE DIFFERENCE METHODS for PDE

Solve the following parabolic PDE

$$\frac{\partial u(x, t)}{\partial t} = \Delta \frac{\partial^2 u(x, t)}{\partial x^2} + ku(x, t)$$

with

$$\Delta = 2$$

$$k = 1$$

$$L = 10$$

$$t_{\text{final}} = 0.5$$

$$\text{IC: } t = 0 \quad u(x, 0) = x$$

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BC:  $C(t) = t$   
 $F(t) = t \cdot \exp(-t)$   
 $A=D=1$   
 $B=E=0$

## Questions

3.1. which type are the **Boundary Conditions**?

3.2. adopt the **explicit method** and, using **MUC**, explain the procedure briefly, attach the graph and comment the final solution

3.3. how much is  $\Delta x$  ?

3.4. once  $\Delta x$  is fixed, how much should **TIMESTEP** be to get  $\text{LAMBDA}=0.5$  ?

3.5. take a double value of  $t_{\text{final}}$ , maintain the **explicit method** and, using **MUC**, attach the graph

3.6. compare the results of the two cases

## Section 4: MATHEMATICAL MODELING

### 4.1 Classification of a model

With ref. to the following model:

$$\dot{x}_1 = 1 + \frac{x_1}{1.1} + \frac{\sqrt{x_1 x_2}}{2.2}$$

$$\dot{x}_2 = 2 + \frac{x_1}{2.1} + \frac{x_2}{2.2}$$

with  $x_1 = x_1(t)$ ;  $x_2 = x_2(t)$

IC:  $x_1(0) = 0$ ;  $x_2(0) = 0$

- a) identify at least one general classification to which this model can be attributed
- b) within the classification you've chosen, provide all attributes and categories you consider pertinent to this model
- c) if the following further eq. is added

$$x_3(t) = x_1(t)^2 + x_2(t)^2$$

does the **model order** change? How much?

## Section 5: TIME SERIES

### 5 Moving average

- a) what are meaning and objective of moving average in **time series** data analysis?

The demand for a product in 7 months is shown by the short **time series** in the file:

sect.5\_Time Series\_2025-07-14.txt

- b) obtain a new **filtered time series** from the above data by adopting a **moving average** with the **span**  $M=4$
- c) plot the new **filtered time series** against the original one and comment them