**Last Name** **Name** **student ID (matricola)**

n = the last one or two digits from the student ID No. (*N. matricola*)

## Section 1: LINEAR PROGRAMMING

A canning company must produce fruit juice by mixing fruit pulp and a sweetener, thus obtaining a final product that must meet certain requirements regarding vitamin C, mineral salts, and sugar content.
The fruit pulp and sweetener are purchased at a cost of 400 lire and 600 lire per hectogram (hg), respectively.
Furthermore, the suppliers ensure that 1 g of fruit pulp contain 1.4 mg of vitamin C, 0.2 mg of mineral salts, and 0.25 g of sugar, whereas 1 g of sweetener contain 0.1 mg of mineral salts, 0.5 g of sugar and no vitamin C.
The requirements for the fruit juice ready for sale are set for 1 hg of the final product.
Therefore, the fruit juice must contain:

- at least 7 mg of vitamin C per hg of final fruit juice
- at least 3 mg of mineral salts per hg of final fruit juice
- and at least 7.5 g of sugar per hg of final fruit juice

Determine the quantities of fruit pulp and sweetener to be used in the production of 1 hg of fruit juice to minimize the overall cost of purchasing the two basic components.

# Questions

1.I. Formulate and write the LP math model of this problem

1.II. What type of LP problem is this❓

1.III. Solve it in Matlab and describe step by step the obtainment of the **optimum**

NB: the student using the Graphical Method will get 1 EXTRA POINT added to the mark

1.IV. Determine the optimal value of the **objective function**

1.V. Determine the optimal values, if any, of the **decision variables**

1.VI. At the **optimum**, provide comments on special or unexpected features, if any

## Section 2: EMPIRICAL MODELS

This section is alternative to **Section 5**: you can solve only one of both❗

In an experiment aimed at measuring vapor pressure, data was obtained in the following file.

sect.2_Pv data_2026-02-27.txt

# Questions

First, carefully look at data and then use Matlab® tools:

2.1. plot the original data

2.2. determine a **regression model** that you consider reasonably valid, taking into due consideration the presence of **ouliers**

2.3. is the regression model adopted by you a LINEAR or NON-LINEAR one?

2.4. calculate and plot the **residuals** of the **regression model**

2.5. discuss the **quality of the residuals** by generating and attaching the **Error Density Plot**

2.6. using the regression model adopted by you, calculate the **Extrapolated point** at an abscissa T=98 °C and discuss it

2.7. Remove the **ouliers** (if any) from the original data set and determine an **interpolation model** that you consider reasonably valid

2.8. discuss the quality of previous **interpolation model** and compare it to the previously obtained **regression model**

## Section 3: FINITE DIFFERENCE METHODS for PDE

Solve the following parabolic PDE $$\frac{\partial u(x,t)}{\partial t} = \Delta \frac{\partial^2 u(x,t)}{\partial x^2} + ku(x,t)$$

with

$\Delta = 2$

$k = 0$

$L = 10$

$t_{final} = 0.5$

IC: $t = 0$ $u(x,0) = 0.5x$

BC: $C(t) = t$

$F(t) = t \cdot \exp(-t)$

A=D= n

B=E=0

# Questions

3.1.which type are the **Boundary Conditions**?

3.2.Choose a reasonable *#Points* and calculate Delta_x

3.3.once Delta_x is fixed, how much should TIMESTEP be to get LAMBDA=0.5 ❓

3.4.adopt the **explicit method** and, using **MUC**, explain the procedure briefly, attach the graph and comment the final solution

3.5.adopt the **Crank-Nicholson method** with the same *#Points* as above; then, using **MUC**, explain the procedure briefly, attach the graph and comment the final solution

3.6.compare the results of the two cases

## Section 4: MATHEMATICAL MODELING

### 4.1 SIR Math model

For the SIR math model:

SIR Lumped Parameter Model

*(see the "SIR Modeling 494-561 Spring 2024..." handout)*

$$\frac{dS(t)}{dt} = \underbrace{\frac{B_r}{\mu}}_{\substack{\text{Birthrate}\\(\text{inflow})}} - \underbrace{\beta(t)S(t)I(t)}_{\substack{\text{Infection}\\(\text{consumption})}} - \underbrace{\mu S(t)}_{\substack{\text{Mortality}\\(\text{outflow})}} = f_1(x,u,d)$$

$$- \underbrace{k_v(t)S(t)}_{\substack{\text{Vaccination}\\(\text{consumption})}} + \underbrace{\rho(t)\left(\frac{B_r}{\mu} - S(t) - I(t)\right)}_{\substack{\text{Loss of Immunity}\\(\text{generation})}}$$

$$\frac{dI(t)}{dt} = \underbrace{\beta(t)S(t)I(t)}_{\substack{\text{Infection}\\(\text{generation})}} - \underbrace{\gamma(t)I(t)}_{\substack{\text{Deactivation}\\(\text{consumption})}} - \underbrace{\mu I(t)}_{\substack{\text{Mortality}\\(\text{outflow})}} = f_2(x,u,d)$$

$$x = \begin{bmatrix} S(t) \\ I(t) \end{bmatrix}, \quad u = [\beta(t)], \quad d = \begin{bmatrix} \gamma(t) \\ k_v(t) \\ \rho(t) \end{bmatrix}, \quad f(x,u,d) = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

a) how much is the **order**❓

b) what are the **state variables** (if any) ❓

c) provide all relevant classification keys, according to the theory of Model Classification, e.g., 1, 2, 3, etc.

## Section 5: TIME SERIES

This section is alternative to **Section 2**: you can solve only one of both❗

With ref. to data in the **time series** within the file:
sect.5_Time-series (AR1) 2026-02-27.xlsx
and the use of Matlab®,
first perform and comment the **exploratory data analysis**:

5.1) Plot the original **time series**

5.2) Calculate mean

5.3) Calculate standard deviation

5.4) Calculate skewness

5.5) Calculate kurtosis

Then work on **autocorrelation**:

5.6) What is the **autocorrelation**❓

5.7) Detect **autocorrelation**, if any, in the above Time Series and plot the **correlogram**

5.8) Propose a reasonable value of the **No. of lags**

5.9) Comment the following cases: