

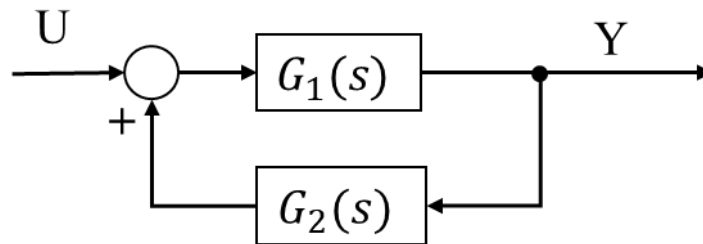
Last name

Name

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## Section 4: STABILITY OF LINEAR DYNAMIC SYSTEMS

A dynamic process  $G_p(s)$  is here considered that is a “recirculative process”<sup>1</sup>:



Where  $U(s)$  is the **manipulated variable**:

$$G_1(s) = \frac{-1.5}{0.5s + 1}$$

and

$$G_2(s) = \frac{1}{2s - 1}$$

and their combination with the block algebra introduces a **positive zero**.

I. What is the **process transfer function**  $G_p(s)$ ?

In order to put the dynamic process  $G_p(s)$  in a feedback control configuration, a unity gain element represents the measuring device and a proportional-integral controller is used:

$$G_c(s) = K_c \left( 1 + \frac{1}{\tau_I s} \right)$$

where  $\tau_I=1$ .

II. What is the **open loop transfer function**  $G_{OL}(s)$ ?

<sup>1</sup> Koichi. Inoya and Roger J. Altpeter, Industrial & Engineering Chemistry 1962 54 (7), 39-43

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## Part A: Root locus

A1. Plot the *root locus* by means of MATLAB or SisoTool and attach it here.

A2. Which rules are applied to build the root-locus, direct or the inverse root locus?

A3. If any, find the  $K_c=K_c^*$  of the controller for which the system in closed-loop is marginally stable.

A design requirement is available for this process, the **percentage of overshoot must be lower than 10%**.

A4. Find  $K_c=K_{c\text{design}}$  of the PI controller for which the system respects the design requirement.

A5. Report here a screenshot of the *root locus* (from SisoTool) for the previous defined value showing the placement of the closed-loop poles (*Hints: it may be useful to look also at question C1 at this point*).

## Part B: Frequency response

Considering the previously found **open loop transfer function**  $G_{OL}(s)$  and with  $K_c=1$  in the PI controller:

B1) Plot the **Bode Diagrams** by means of MATLAB or SisoTool resources and attach it here

B2) Does a **crossover** frequency exist? How much is it?

B3) Decide if the Bode stability criterion is applicable.

B4) If yes, is the above system closed-loop stable?

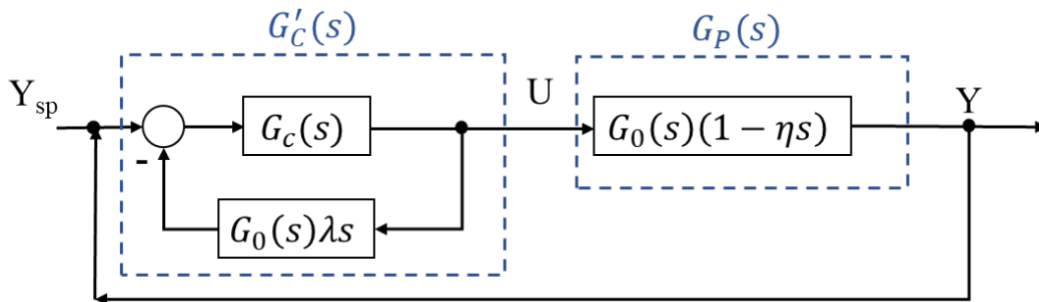
B5) Plot and attach here the **extended Nyquist diagram** *together with the unit circle* by means of MATLAB or SisoTool resources, then discuss it in detail.

B6) Check, on the base of the **Nyquist** stability criterion, if the above system is closed-loop stable

## Inverse Response Compensator

- I. Find a rational transfer function  $G_0(s)$  such as that  $G_P(s) = G_0(s)(1 - \eta s)$  where  $(1 - \eta s)$  represents the position of the positive zero of the original  $G_P(s)$

According to “Inoya and Roger”, an **inverse response compensator** can be built using the following feedback architecture:



where  $\lambda$  is an optimizable parameter, that can be set to  $\lambda = 2\eta$  for optimal results<sup>2</sup>.

- II. How much is the value of  $\lambda$ ?
- III. Write the new transfer function of the **compensated controller**  $G_c'(s)$ , using for  $G_c(s)$  (PI controller) the value of  $K_c = K_c^*$  previously found.

## Part C: Dynamic responses in the time domain

Considering the feedback control system with the controller block given by the **uncompensated PI controller**  $G_c(s)$ , with the gain  $K_{c\text{design}}$  previously determined

- C1. Plot the **closed-loop** system dynamic response to a unit step in **set point** by means of MATLAB or SisoTool resources with the rise time and peak response and attach it here.

Considering the feedback control system with the controller block given by the **compensated controller**  $G_c'(s)$ , with the gain  $K_c = K_c^*$  previously determined

- C2. Plot the **closed-loop** system dynamic response to a unit step in **set point** by means of MATLAB or SisoTool resources with the rise time and peak response and attach it here.
- C3. Does the inverse response compensator improve the closed-loop stability of the system? Why?
- C4. Does the inverse response compensator maintain or remove an inverse response pattern in the time domain?

<sup>2</sup> Ray, W. H., Ogunnaike, B. A. (1994). Process Dynamics, Modeling, and Control: Oxford University Press.

**Part D:**

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