**Section 6:** **MATHEMATICAL MODELLING**

A heated and stirred mixer for liquids (see figure) is fed from top by 2 streams, the first one with volumetric flow rate 1 at temperature T1 and another with volume flow rate 2 at temperature T2. The output is from the bottom with volumetric flow at temperature T.

The mixer is made of a cylinder (diameter D and height H) placed above an inverted cone (see figure) with the same cross section.

The lateral surface of the tank is equipped with a jacket exchanging heat with a global exchange coefficient US; the heating fluid, which circulates in the jacket, is steam that completely condenses at pressure PS. The surface of the lower cone is exposed to the outside air at temperature Ta, it is not insulated and therefore disperses heat with a global exchange coefficient Ua.



The following hypotheses hold:

1. perfect mixing in the container
2. 1 = constant
3. 2 = constant
4. ρ = constant
5. hs (height of the liquid in the container) = H = constant

You must:

1. Write a **steady state** model
2. Write a **dynamical** model
3. **classify** the obtained dynamical model
4. list **input, state, output variables** and the **parameters** of the model
5. discuss which input variables can be assumed as **forcing functions** and which are their possible functional forms for the physical feasibility
6. re-write the model using the **deviation variables**
7. re-write the model using the **canonical form** for it

As another case study, you are asked to consider Ua as a function of temperature according to following Eq: Ua=k T1/2.

1. discuss what changes will occur in the dynamical model.

As a further case study, you are asked to consider the case in which the stream with volume flow rate 2 is made of melting ice entering at constant temperature T2 = 0 °C.

1. discuss what changes will occur in the dynamical model.

SOLUTION

***Hypotheses:***

1. Perfect mixing

2.

3.

4 hs (height of the liquid in the container) = H = constant,

5. Ps = constant and Tc = constant (for the condensing steam)

**Steady-state Mathematical Model:**

IN – OUT + GEN = 0

Mass balance:

eq. (1)

Energy balance on the system:

where Tr is a generic reference temperature for liquid water.

When considering eq. (1) and replacing the volumes we have:

By rearranging:

eq. (2)

N.B.: in the following developments it is simpler to consider the entire Vtot volume, rather than making it explicit in terms of H and D.

**Dynamical Mathematical Model:**

Enthalpy balance:

IN – OUT + GEN = ACC

considering eq. (1), and replacing the volumes we have:

eq. (3)

subject to C.I.: t = 0 T(0) = Ts

**Classification of the Dynamic Mathematical Model:**

• Macroscopic

• described by an ODE, first-order, linear, non-homogeneous, at constant coefficients

**Mathematical model in terms of deviation variables:**

Subtracting eq. (2) from eq. (3) we obtain the expression of the mathematical model representing the system in deviation variables:

eq. (4)

subject to C.I.: t = 0 T’(0) = 0

Defining the constants as follows:

time constant

static gain of the first inlet

static gain of the second inlet

eq. (4) can be expressed as follows:

eq. (5)

## Identification of input, status and output variables, as well as model parameters:

Input variables: T1, T2

State variables: T

Output variables: T

Parameters: Ua, Us, Ps, Ta, H, D, cp, ,.

N.B.: this system has two input forcing functions, T1 and T2, which can be independent of each other.

## Input variables that can be taken as forcing functions and their type:

T1, T2  could be of the oscillatory (at low frequency) or limited ramp type.

## Dynamical Mathematical Model for the case h):

In this case it is: Ua = k T0.5, therefore for the system will have:

eq. (6)

## Classification of the new Dynamic Mathematical Model:

* Macroscopic
* Eq. (6) constitutes a non-linear, non-homogeneous ODE, with constant coefficients.

## Dynamical Mathematical Model for the case i):

Enthalpy balance:

IN – OUT + GEN = ACC

where is the latent heat of fusion per unit mass.